Context-Dependence of Mathematical Activity: A Case Study Concerning Edo Period Japan

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Rosalie Joan Hosking

University of Canterbury

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ABSTRACT

At the beginning of the Edo period (1600-1868 CE) the Japanese Tokugawa *shōgunate* enforced the famous closed country policy. During the period of isolation that would ensue until the Meiji Restoration, mathematics flourished like never before. The new tradition that arose was rich and diverse, with mathematics manifesting itself through different practitioners in many different ways. And, for the first time in Japanese history, mathematics began to diverge from Chinese practice, developing a uniquely Japanese identity.

Because of this, we therefore can look to Edo mathematics with the expectation that it can especially clearly illustrate cultural variability in the practice of mathematics if it is the case that there exists such.

The present thesis examines whether cultural-contextual factors from within the isolated Edo environment impacted individual practitioners of mathematics to result in the variation and uniqueness that appeared. Also, it highlights and addresses what the consequences might be for historians, philosophers, and mathematicians if such an influence did occur.

PREFACE

My objective in this research is to encourage deliberation regarding the importance of context historically for mathematics. This task is done through an examination of the impact context had on Japanese mathematics of the Edo period. An additional goal is to illustrate how this influence prompts the re-evaluation that social constructivist and realist interpretations of mathematics are not mutually exclusive.

The influence of context on Edo period mathematics is herein illustrated through three case studies, each of which examines a separate occurrence of mathematical practice in the era. The first details the ways in which context shaped Yoshida Mitsuyoshi's text the *Jinkōki.* The second investigates Takebe Katahiro and his *Tetsujutsu Sankei.* And thirdly and finally, the mathematical tablets of the *sangaku* tradition are examined.

I conclude that context played a role in influencing the development of each of these instances of mathematics, as well as the differences between them. By also examining examples from other traditions past and present, I furthermore conclude that this influence by context on mathematical practice is by no means limited to these Edo cases. I therefore argue that context plays an important role in shaping the development of mathematics generally.

I argue however that this evidenced connection between mathematical development and context does not exclusively support the thesis of social constructivism. While Edo mathematics was impacted by contextual factors, results and methods within other various traditions are noted which are either similar or comparable in sophistication and style to instances in Edo Japan. This may support both social constructivism and universalism, and thus call for a re-evaluation of traditional interpretations of mathematics.

Acknowledgements

I would like to thank my supervisors Philip Catton and Clemency Montelle for their expertise, guidance, and patience during the course of this research. Without their continued support and insights this thesis would not have eventuated.

I would also like to thank Dr Hiroshi Kotera for granting me permission to use photos of various *sangaku* tablets of his collecting.

Thanks is also due to my friends and family, who gave me constant encouragement and support while this research was produced.

GLOSSARY

Bakufu/ Shōgunate 幕府– feudal rulers of Japan.

Daimyō 大名 – local warlords who governed over various prefectures.

Edo 江戸 – the capital of Japan from 1600 CE onwards, now known as Tokyo.

Ema 絵馬 – shrine and temple offerings.

Jinkōki 塵劫記 – popular Edo period mathematical textbook of Yoshida Mitsuyoshi.

Kanbun 漢文 – an academic language which used Chinese characters.

Ri 理 – the organising principle of Neo-Confucianism.

Sankin Kotai 参勤交代 – the alternative attendance policy enforced in the Edo period.

Sangaku 算額 – geometrical mathematical tablets of the Edo period.

Sangi 算木 – Japanese mathematical counting rods.

Shōgun 将軍 – the feudal military leader of Japan.

Soroban 算盤 – Japanese abacus.

Suanfa Tongzong 算法統宗 – famous Chinese mathematical text of Cheng Dawei.

Sūri 数理 – organising principle related to mathematics.

Tetsujutsu Sankei 綴術算経 – Edo period mathematical text of Takebe Katahiro.

Tokugawa 徳川 – ruling *shōgunate* family from 1600 to 1868 CE.

Wasan 和算 – traditional Japanese mathematics developed during the Edo period.

PERIODS IN JAPANESE HISTORY

35,000 – 14,000 BCE	**Palaeolithic**
14,000 – 300 BCE	**Jōmon**
300 BCE – 250 CE	**Yayoi**
250 – 710 CE	**Yamato**
710 – 794 CE	**Nara**
794 – 1185 CE	**Heian**
1185 – 1333 CE	**Kamakura**
1333 – 1573 CE	**Muromachi**
1573 – 1600 CE	**Momoyama**
1600 – 1868 CE	**Edo/Tokugawa**
1868 – 1912 CE	**Meiji**
1912 – 1926 CE	**Taishō**
1926 – 1989 CE	**Shōwa**
1989 – Present	**Heisei**

CHAPTER 1

EDO PERIOD CONTEXT AND YOSHIDA MITSUYOSHI'S MATHEMATICS

INTRODUCTION

During the Edo period context was defining of the development of mathematics and manifested itself in different ways through different people as the nation struggled to assert and define its own identity. The mathematics of Yoshida Mitsuyoshi 吉田 光由 (1598-1672 CE) – a figure appearing at the beginning of the era who greatly inspired the indigenous *wasan*[1] 和算 tradition – was heavily instructional and utilitarian due to influence from external contextual factors. However, additional factors saw his mathematics also contain problems of complexity that seemed to reach towards the supra-utilitarian. Mathematics of later practitioners would develop in different ways again to be more abstract, religiously affected, and of a stronger recreational nature (depending on the practitioner).

Yoshida Mitsuyoshi's mathematics, as shall be seen, shows the importance of context for Edo mathematics. This is because it is dependent upon and influenced by certain cultural, social, and political factors for its form, content, demographics and purpose. Also, the ways in which it was shaped by context differed to the mathematical work of later practitioners, as shall be later evidenced.

[1] *Wasan* is the term given to the indigenous Japanese mathematical tradition developed during the Edo period, where *wa* (和) stands for Japan and *san* (算) calculation.

One contextual element for instance that impacted Yoshida Mitsuyoshi's text the *Jinkōki* (塵劫記) – first published in 1627 CE, with new editions in 1629, 1631, 1634, June 1641 and November 1641 CE – that would not be as important for later mathematicians was the economic climate of the beginning of the Edo period. This particular climate saw an increased need for instructional mathematics regarding commercial activities develop which Yoshida can be seen to directly respond to.

Another substantial development was the introduction of the Japanese version of the abacus. The abacus would prove to be a useful tool for commercial activity and happened to appear in Japan just prior to the beginning of the Edo period. Other conditioning factors to impact Yoshida's mathematics also included the banning of foreign books, the introduction of the isolation policy, the prior study of Chinese mathematics in Japan, the Japanese tendency to adapt Chinese knowledge and culture into something Japanese, the class system, and the popularity of the *Jinkōki* text itself.

In this chapter, each of these factors and how they impacted and shaped the mathematics of Yoshida's *Jinkōki* are discussed.

1.1. Economic Growth

One of the major influences to impact the form, content, and purpose of the mathematics in Yoshida Mitsuyoshi's *Jinkōki* was the intense economic growth that occurred during the beginning of the Edo period. In this section, how certain features of the *Jinkōki* were dependent upon this social change will be shown. It is argued that this impact evidences that context was defining of and important for the development of Yoshida's mathematics.

Edo, Alternative Attendance, and Commerce

The Edo period began after the first of the Tokugawa *shōguns* 将軍 – Ieyasu Tokugawa 徳川 家康 – rose to power and brought unity and peace to the country in 1600 CE after a "long lasted period of civil wars".[2] The new *shōgunate* feudal regime instigated many new changes and policies, the most important and influential being the shifting of the capital from Kyoto to Edo (modern day Tokyo), the national seclusion policy 鎖国[3], and the *sankin kotai* 参勤交代 alternative attendance policy of 1635 CE.

Before the Tokugawa *shōgunate* shifted the capital and government to Edo, the city had been "a minor fishing village of little significance".[4] However, "by 1700 it had perhaps a million residents" due to the extensive growth which occurred in the centre as a result of this political decision.[5]

Its growth was also the result of the alternative attendance policy which had forced the families of all *daimyō* 大名 (local lords) to permanently reside in the new capital and *daimyō* themselves to personally spend six months of every year there. Prior to the official enforcement of the law, many *daimyō* already willingly travelled to Edo on a frequent basis to show their support for the new *shōgunate*. Some also preemptively left family members there as a sign of good will, indicating how the growth of the capital began almost immediately following the change in government.

Following officials and the family of *daimyō* to Edo, "among its first new residents…were merchants from Mikawa and Tōtōmi, provinces which had once

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[2] Osamu Takenouchi, *Jinkōki*, p. 14

[3] This policy was largely in response to the growing number of foreign missionaries, in particular Christian, who were entering the country and starting to influence local daimyo. Ieyasu had feared that converted *daimyō* might join together and conspire against him, threatening his position as *Shōgun*.

[4] David Flath, *The Japanese Economy*, p. 23

[5] Ibid.

been ruled by Ieyasu. After them came men from Ōmi, Ise, and Osaka, who opened markets for their own special products. Thus trade began to flourish”.[6]

The sudden and rapid increase in Edo’s population brought on by these events had a dramatic impact on the local and national economy, as:

> …commodities of every sort were funnelled to the center…The provision of materials needed for life at the capital and transporting them there provided economic opportunities for commoners, and as the merchant and artisan classes grew in size and importance a new popular culture emerged.[7]

Due to these factors, Edo became “a centre of wholesale and retail trade on a grand scale”.[8] The economy of Japan began to experience a period of rapid growth, and the merchant and artisan classes grew and blossomed in response to the increased need for goods and services occurring.

The impact this had on the development of mathematics was profound. In order to “perform buying and selling, people needed to make calculations”, and to do so the need of merchants to “use the *Soroban* in practical cases…an urgent necessity”.[9] It was due to these occurrences that Yoshida Mitsuyoshi “made up his mind to write a good text which might be useful to people”.[10] His publishing of the first volume of the *Jinkōki* in 1629 CE came only twenty-nine years after the moving of the capital and government to Edo. It was also six years before the alternative

[6] George Sansom, *A History of Japan: 1615-1867,* p. 114
[7] Marius B. Jansen, *The Making of Modern Japan,* p. 128
[8] Sansom, op. cit., p. 114
[9] Ibid.
[10] Ibid.

attendance policy was officially enforced (though, as discussed, it was unofficially enacted and expected prior to then). The influence of the economic expansion caused by these political decisions is clearly exampled in the instructional nature and commercial content of Yoshida's work, which shall now be discussed.

Instructional Commercial and Agricultural Mathematics

The majority of the mathematics in the *Jinkōki* is of an instructional manner. The work is also largely devoted to dealing with problems of a commercial and agricultural nature, and books I and II in particular have a strong focus on these areas.

In book I, the first nine topics deal solely with instructional mathematics – teaching the reader: the naming of large numbers, naming of small numbers, units of volume, units for the area of rice fields, weight of various substances, the multiplication table, *Hassan* (division on the abacus), *Ken-ichi*, and multiplication instead of division for calculation on the abacus. The rest of the topics of book I are also of a definite commercial and agricultural nature, and include the subjects: trading rice, rice bags, piling up of rice bags, rice bags in a granary, buying silver, exchange of silver, exchange of gold and silver, *Koban* and silver exchange, interest on a loan, price of silk and cotton cloth. These topics are approached in an instructional manner as well, with the mathematics framed such that it could be applicable to everyday situations.

The subject matter and style of the text indicates that the purpose of the mathematics was as a utilitarian tool for commerce and exchange. For example, the following problem from the chapter 'price of silk and cotton cloth' evidences the combination of instructional and commercial mathematics commonplace in the

Jinkōki, and shows how aiding commercial calculation was important for Yoshida:

> 1 *shaku* by the cloth measure is 1 *shaku* 2 *sun* by the carpenter's measure.
>
> 1 *tan* of cotton cloth[11] costs 4 *monme* 5 *bu* of silver. 1 *tan* of cotton cloth is 2 *jō* 5 *shaku* in length. How much does 1 *shaku* of cotton cloth cost? 1 *shaku* of cotton cloth costs 1 *bu* 8.
>
> Process: Divide 4 *monme* 5 *bu* by 2 *jō* 5 *shaku*, and you get 1 *bu* 8 as the value of 1 *shaku*. Another procedure: multiply 4 *monme* 5 *bu* by 4, and you get 1 *bu* 8 as the answer. This way of multiplying by 4 is always convenient when it is required to divide by 25.[12]

As can be seen, this problem deals with commerce and exchange while being of an instructional nature. It presents a problem relating to commerce and the solution and best ways for calculating it are provided. The problem is directly applicable to everyday problems and could be used in real life situations given its dealing with goods, units, and currency of the Edo period. It is the case for example that during the later part of the period the cost of 1 *tan* of cloth from Harima averaged between 5 and 7 *monme* of silver.[13] This indicates that the quantities and values given by Yoshida

[11] A *tan* was the unit for cloth during the Edo period.
[12] Takenouchi, op. cit., p. 85
[13] William B. Hauser, *Economic Institutional Change in Tokugawa Japan: Osaka and the Kinai Cotton Trade*, p. 110

were in fact realistic for actual commercial activity during this time, and further evidences a utilitarian element.

In book II other commercial topics are presented in a similar format. An example can be seen in this problem from the chapter ‘trade in lumber’:

> There are 400 square beams with 3 *sun* sides that are 2 *ken* long. A man wants to exchange these with square beams with 4 *sun* sides that are 2 *ken* long. How many can he get? He gets 225 square beams with 4 *sun* sides.
>
> Process: Square 3 *sun*. It is 9. Multiplying 400 by this, you get 36. On the other hand, square 4 *sun*. It is 16. Divide 36 by this. One knows thus that he can get 225 beams.[14]

These two problems both evidence a connection to commerce in the mathematics of the *Jinkōki*. They also are just two examples of the multiple problems of this manner included in the text.

The content of the *Jinkōki* can be therefore understood as partly determined by the rapid commercial growth occurring at the start of the Edo period. This is because problems from the text such as those examined above clearly respond to the need for mathematical training regarding commercial subjects among the merchant and artisan classes who did experience increased business and made up “more than 80 percent of

[14] Takenouchi, op. cit., p. 121

the total population" of around twelve million at the time.[15] This text was also one of the very first original Japanese mathematical texts produced, and its appearance at a time when Japanese citizens required commercial mathematical training is telling of its purpose.

Also, it is the case that Yoshida's *Jinkōki* was "widely read and…one of the best sellers in the Edo period".[16] There were even "several thousand copies…printed in the Kan'ei era (1624-1644) alone", not to mention the years following.[17] Also, as many as three-hundred books named after it which often contained plagiarised problems from the original text were published during the era.[18] Because of this, the work is claimed to have "played a prominent role in the diffusion of elementary mathematics among the people throughout the Edo era".[19] It seems no coincidence that an original Japanese work with instructional commercial mathematics was so popular during a time in which merchants, artisans and farmers had increased business and a need for useful mathematical training because of the economic climate.

The inclusion of commercial mathematical instructions combined with the fact the textbook became so vastly popular immediately after its publishing indicates that it was indeed the economic boom which the textbook and Yoshida were responding to. Therefore, much of the content of the *Jinkōki* can be seen as being shaped by context, and the commercial mathematics of this text one of the ways in which context expressed itself in the mathematics of individual practitioners in the Edo period.

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[15] Chie Nakane, 'Tokugawa Society', *Tokugawa Japan: The Social and Economic Antecedents of Modern Japan*, p. 215
[16] Kenji Ueno, 'From Wasan to Yozan', p. 69
[17] Masayoshi Sugimoto and David L. Swain, *Science & Culture in Traditional Japan*, pp. 206, footnotes
[18] Ueno, op. cit., p. 69
[19] Tamotsu Murata, 'Indigenous Japanese mathematics, Wasan', p. 105

Soroban

Another way in which Yoshida's mathematics seems to have been influenced by the economic climate is in its use of the Japanese abacus for calculation.

Before the Edo period, Japanese style counting rods known as *sangi* 算木 (adapted from Chinese rods) were used exclusively for calculation. However, in the years prior to the Edo period an altered form of the Chinese abacus known the *soroban* 算盤 began to be produced and distributed amongst citizens.

The Chinese abacus – known as the *suanpan*算盤 – is thought to have found its way to Japan from mainland China in the late 1500s CE.[20] There are rumours that it was in fact Yoshida's teacher – Mōri Shigeyoshi毛利重能 – who travelled to China to learn mathematics and brought back the abacus to the Japanese.[21] However, this is not commonly believed to be historically accurate.

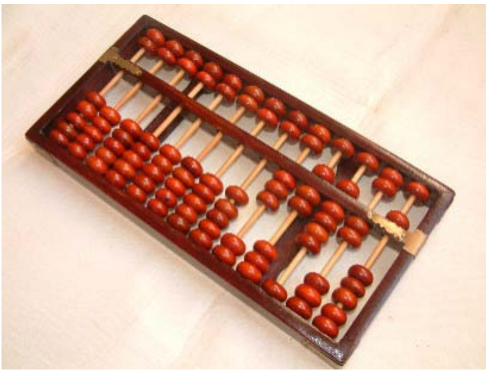

Image 1 - Chinese suanpan

[20] Takenouchi, op. cit., p. 28
[21] Smith and Mikami, *A History of Japanese Mathematics*, pp. 32-3

Upon its entry into Japan, the Chinese *suanpan* was altered. The Japanese opted for one bead instead of two in the upper section (which signify the quantity of five), and rather than having five beads in the lower section (of the quantity one) instead they used four. The beads were also of a grooved shape rather than circular, which greatly aided in the speed and ease with which one could do calculations on the device. It was also more compact and contained more rows, allowing for the calculation of very large numbers.

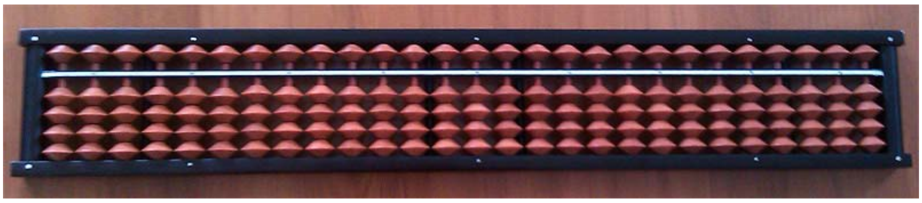

Image 2 - Japanese soroban

Yoshida's master – while perhaps not being the original introducer of the device – was the first to popularise the *soroban* as a tool of calculation. In 1622 CE Mōri published the *warizan-sho*割算書 – a work sometimes considered "the first Japanese literature on mathematics" – which was a treatise dedicated to use of the new device.[22]

Yoshida also used the *soroban* over *sangi* rods in the *Jinkōki*. This was likely due to his being instructed in its use by his master, the fashionable status it was attaining, and the fact that the Japanese version allowed for particularly quick and

[22] Tuge Hideomi, *Historical Development of Science and Technology in Japan*, p. 31

easy calculation – making it perfect for commercial use. During the economic boom, merchants, artisans and farmers did indeed adopt and use the *soroban*, and a need for training in use of the device developed. It soon became that "reading, writing, and abacus skills were critical for the handling and analysis of business information" among these classes.[23] The following saying of the period also illustrates its importance: "While the use of the abacus is one of the most important things a merchant must learn, he should not take it too seriously. Excessive study will hurt business".[24] This indicates that merchants did use the *soroban* for business purposes, and a real need for instruction in the use of the device occurred during the beginning of the Edo period.

Given that Yoshida specifically provides instructions on how to use the *soroban* (for instance in the *Hassan* and multiplication instead of division on the *soroban* sections) and also provides detailed visual step by step instructional guides for many calculations (such as square and cube root extraction) it is likely he did purposely use the *soroban* for calculation as a means to train those who wanted to use the device for everyday and commercial purposes.

It is the case that the *soroban* was to become so associated with commerce and agriculture that some later Japanese mathematicians purposely reverted back to using *sangi* rods. *Samurai,* who made up the majority of mathematicians during the first half of the Edo period and were not allowed to engage in commerce are said to have "despised the plebeian *soroban,* and the guild of learning sympathized".[25] This dislike for the device was likely driven by a wish to disassociate and distinguish

[23] Katsuhisa Moriya, 'Urban Networks and Information Networks', *Tokugawa Japan: The Social and Economic Antecedents of Modern Japan*, p. 118
[24] Shigeru Nakayama, 'Japanese Scientific Thought', *Dictionary of Scientific Biography XV,* p. 747
[25] Smith and Mikami, op. cit., p. 47

themselves from the lower classes where *soroban* use was flourishing. With regard to the guild of learning, a wish to establish a difference in quality and ability between the mathematical activity they engaged in and that of the lower classes may have driven this negativity towards the calculation tool. This hints once more at the popularity the device had amongst the lower classes who had a need for calculation for business purposes.

For these reasons, it is likely that Yoshida was impacted by additional contextual factors – namely the introduction and adaption of the *soroban* – in his use of the *soroban* as the preferred tool of calculation in the *Jinkōki*. This was due in particular to the increasing popularity of the *soroban* among the merchant, farming, and artisan classes and their need for mathematical training due to the economic growth occurring at the beginning of the Edo period. Also, prior to the *Jinkōki,* the only text dealing with *soroban* instruction was the *warizan-sho* of Yoshida's master Mōri. This indicates that there was a lack of instructional material on the *soroban* during the period, and because of the economic growth and need for *soroban* instruction Yoshida adopted the *soroban* rather than *sangi* rods for calculation. This thus examples another aspect of his work influenced by context.

Summary

For the reasons stated above, it can be seen that there was a significant impact from the economic climate on the contents and method of calculation in the *Jinkōki*. The content of this work takes the form it does – as highly instructional and commercial in the first two books – because of factors such as the growth caused by the shifting of the government to Edo and the alternative attendance policy. Also, the recent distribution and adoption of the *soroban* and its usefulness for calculation during this period of growth influenced the inclusion of instruction pertaining to this device and use of it for calculation rather than *sangi* rods.

1.2. Alternative Attendance

As mentioned, at the beginning of the Edo period many local *daimyō* lords showed their support for the new *shōgun* Ieyasu Tokugawa and his family by personally visiting him in Edo. They also sent "family members to Ieyasu in Edo as hostages" or insurance to show their allegiance.[26] As well as helping to create an increase in commerce, this action also impacted business transactions due to different regions using different currencies and most goods coming from the Kansai region.[27] This created a particular need for mathematics pertaining to currency conversation which can be seen to be addressed in the *Jinkōki.* The alternative attendance policy thus impacted the type of commercial problems included in the text, and the mathematics of Yoshida in general.

Yoshida Mitsuyoshi was born two years before Ieyasu Tokugawa came to power and shifted the government to Edo. By the time he was thirty-seven years old in 1635 CE, Ieyasu's grandson Iemitsu Tokugawa 徳川 家光 had enforced in law the alternative attendance policy making regular trips to Edo compulsory for all *daimyō*. Because of this, in the time in which Yoshida lived currency conversion was particularly important for both officials and merchants. This was because *daimyō* and their families from the Kansai region brought silver with them during their relocation

[26] Jansen, op. cit., p. 129. Family members were sent to permanently live in Edo to ensure that *daimyō* would stay loyal to the new ruling family, for if a *daimyō* did not obey the Tokugawa their family would be at risk. Before it was made law for their families to reside in Edo, many chose to show their loyalty by sending family members to the capital willingly.
[27] The Kansai region includes the Osaka-Kyoto-Nara region southern to Tokyo.

and frequent visits to Edo which would require converting into the local currency. Edo merchants also needed to buy in silver if purchasing goods from Kansai.

The currency system of the time was complex due to different regions and classes each using different currencies in this manner. Takenouchi writes:

> The currency used in Edo was gold, while that used in…Kyoto – Osaka area, was silver. Among the general public, copper coins were circulated. So the exchange of these difference sorts of money was not an easy matter….The conversion among them much troubled people.[28]

This need for currency conversion can be seen to directly shape the form of many commercial problems found in the *Jinkōki.* As discussed in the last section, the inclusion of commercial problems in the text can be considered a result of the economic boom occurring in Japan during the time Yoshida lived. However, the specific content of some of these problems can be explained by other external influences. The alternative attendance policy and the complicated currency system example such influences that directly impacted the content of specific problems which deal with the issue of conversion.

One example for instance which seems directly impacted by these occurrences comes from the chapter 'Exchange of Gold and Silver'. In this chapter Yoshida details how to calculate the amount of silver one obtains from a quantity of gold given a certain market exchange rate. Problems two and three from the chapter are listed below:

[28] Takenouchi, op. cit., p. 15

> There is 7 *monme* 4 *bu* 8 of gold. A man exchanges this with silver. One *Ohban* at the market exchange rate equals 500 *monme* of silver. If he first divides 500 *monme* by 44 *monme*[29], then it is known that 1 *monme* of gold equals 11 *monme* 3 *bu* 6363 of silver. If he multiples this by the said amount 7 *monme* 4 *bu* 8 of gold, then he gets the following answer.
>
> 84 *monme* 9 *bu* 9995 of silver. One will see that this way of calculation is inadequate. See the next.
>
> In the previous case, if one wants to know how much silver is equal to 7 *monme* 4 *bu* 8 of gold, the exact answer is this. 85 *monme* of silver. **Process:** If you multiply 7 *monme* 4 *bu* 8 of gold at the market price of 500 *monme*, then you get 374. Divide this by 44 *monme*. Then you have 85 *monme*.
>
> This is a good calculating method. This method should be remembered on all occasions.[30]

The mathematics here is concrete, practical, and commercially applicable. This particular problem shows two ways of calculating the amount of silver one can get from 7 *monme* 4 *bu* 8 of gold, with one method specifically emphasised as better

[29] 44 *monme* of gold was a standard equivalent of 1 *Ohban* of gold.
[30] Takenouchi, op. cit., p. 79

than the other. In the second method, which Yoshida tells us is best to use, the amount of gold for conversion is multiplied by the exchange rate (1 *Ohban*) in terms of silver (500 *monme*). The product is then divided by the amount of gold in terms of *monme* that the exchange rate equates to (44 *monme*).

We are told by Yoshida with respect to the first method that "Novices often make the calculation as in the above, being unaware that it is inapt and can lead to unrealistic or awkward incorrect results".[31] The first result does provide an answer that is particularly awkward and unrealistic, for one must produce 9995 coins (of *rin*) if doing a transaction this way, whereas the second form allows for an easier exchange in a round figure. This calculation would also be more difficult to produce on the *soroban* and take more time given the awkward numbers.

The context of this problem combined with its instructions aimed towards the best and easiest form of calculation for merchants indicates this problem was also a response to the economic boom. However, this particular type of commercial mathematics was also specifically responding to the need for currency conversion that occurred due to the increased travel the alternative attendance policy caused as well as the subsequent mixing of currencies it brought.

Thus, these kinds of problems were dependent upon external political and social circumstances as well as the economic climate for their content and inclusion in this mathematical text.

[31] Ibid.

1.3. Rice Economy

Another contextual influence that led to specific types of economic problems appearing as themes in the *Jinkōki* was the abundance of rice and its status as the most important commodity during Edo times.

In Edo Japan "society was organized around a rice-based economy in which agricultural productivity was the principle measure of wealth".[32] The status of *daimyō* for instance was determined by the amount of *koku* they had (where 1 *koku* roughly amounted to the quantity of rice required to feed one person for a year). A *daimyō* usually was "defined as a feudal lord…with an area assessed at the level of 10,000 *koku* or higher".[33] Officials and *samurai* were most often paid directly in rice, and from this they would trade for currency and goods. Rice paddies were also the main crop of farmers, meaning that almost every part of society high and low dealt with the commodity in some way for their livelihood and status.

Yoshida Mitsuyoshi's commercial and agriculturally applicable problems reflect this importance, for he provides a wide range of instructional commercial problems dealing with trading in rice as well as calculating quantities of it (i.e. the problems 'Rice Bags', 'The Piling of Rice Bags', 'Rice Bags in a Granary').

For example, in book I of the *Jinkōki* there exists a chapter entitled 'trading rice'. It contains eleven problems relating to the exchanging of currency for rice. Problem three is listed below:

> A man has 13 kan 485 monme 2 bu 5 of silver. When the price of
> 1 koku of rice is 23 monme 7 bu 5, how much rice can he get with

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[32] Yōtarō Sakudō, 'Management Practices of Family Business', from *Tokugawa Japan: The Social and Economic Antecedents of Modern Japan,* p. 147

[33] Jansen, op. cit., p. 38

the money he has? 567 koku 8[34] to of rice.

Process: Divide the amount of silver, 13 kan 485 monme 2 bu 5, by the price, 23 monme 7 bu 5, and you have the quantity of rice.

The quotation of rice. Note: Money divided by the quotation gives the quantity. Quantity multiplied by the quotation gives the money.[35]

This particular problem details how one can exchange a specified quantity of silver – one of the currencies used during the period in the Kansai region – for rice. It is one of nineteen problems which specifically deal with trading, counting, stacking, and storing rice. It reflects the illustrated importance of this commodity for citizens in the Edo period.

It is the case that some of these problems seem to reach into the realm of the supra-utilitarian however, and may be the result of Yoshida naively assuming certain problems would be useful for farmers when the reality of farming practice meant their usability would have been rare.

For example, in 'The Piling up of Rice Bags' Yoshida details how to calculate the number of rice bags in a heaped pile, with one problem dealing with the number of bags when the first layer totals 13 bags, the top layer 1 bag, and the pile is 13 layers high.[36] While this kind of problem could definitely be of use to farmers, in most situations the farmer would already have the amount of rice bags in their possession on record, and one would assume that when piling the bags they would

[34] Note: 1 *koku* = 10 *to,* 1 *to* = 10 *shō,* and 1 *shō* = 10 *gō*.
[35] Takenouchi, op. cit., p. 65. The calculation is the equivalent of 1348525 x 2375 = 567.8.
[36] Takenouchi, op. cit., p. 71

take count of the number. These problems then, while still applicable to agricultural situations, do appear slightly less immediately useful than other problems and have an air of leaning to the supra-utilitarian.

But, nonetheless, these problems can be understood as included due to the economic boom like those regarding currency conversion, and take the specific form they do because of external contextual influences. In this case it was the high importance and value of rice in the Japanese economy that saw their inclusion. Thus the content of these problems can be seen to be largely determined by the importance of rice in Edo Japanese society.

1.4. Chinese Influence

Yoshida Mitsuyoshi's *Jinkōki* was also dependent upon context due to its being partially influenced by Chinese mathematics. In particular there is evidence that a transmission of knowledge from the Chinese mathematician Cheng Dawei's程大位 1592 CE text the *Suanfa Tongzong* (or *Sanpō Tōsō*) 算法統宗 and the Chinese classic the *Juizhang suanshu* 九章算術 or *Nine Chapters on the Mathematical Art* (hereafter referred to as the *Nine Chapters*) occurred. It is the case that "Japanese mathematicians of the Edo period did not have access to the Nine Chapters" directly per se.[37] However, much of the content of Cheng Dawei's *Suanfa Tongzong* was

[37] Mitsuo Morimoto, 'The Counting Board Algebra and its Applications',数理解析研究所講究, 第 1648 2009, p. 173

based off this early text and in “9 of the 17 chapters...the topics of the 9 chapters” are repeated.[38]

In this section, the dependence upon the prior study of Chinese mathematics in Japan on Yoshida’s work will be shown. First the square root extraction methods of China will be looked at and compared with the method found in Yoshida’s work. It will be shown how there is a correlation evident between these methods, and thus a transmission of knowledge. After this the ‘difficult problems’ included in Cheng Dawei’s work and their connection to the *idai* problems found in the November 1641 CE edition of the *Jinkōki* will be briefly discussed.

While some Chinese methods will be shown to have shaped and determined the content of the *Jinkōki*, it is the case that the methods in the text have been altered and do differ in many ways. Therefore it is not argued that they are solely Chinese methods, but rather Japanese methods which were shaped and inspired by them.

Chinese and Japanese Square Root Extraction Methods

Methods of root extraction in China date back as far as the Han dynasty between 206 BCE and 221 CE. One of the earliest forms can be found in the mentioned *Nine Chapters*. The importation and use of the Chinese *Nine Chapters* in Japan dates back to at least 701 CE, when Emperor Mommu文武天皇established the first University system of Japan.[39] During this time, Chinese mathematical classics such as the *Nine Chapters* were taught.[40] Students also learned how to use the

[38] Frederick K. S. Leung, *Mathematics education in different cultural traditions: a comparative study of East Asia and the West* , p. 97
[39] Smith and Mikami, op. cit., p. 9
[40] Shigeru Jochi (2000), ‘The Dawn of *Wasan* (Japanese Mathematics)’, pp. 425

Chinese bamboo counting rods known as *ch'eou* or *suanzi* 籌 (which *sangi* would be developed from).[41]

With regard to the method of the Ancient Chinese, to "extract the square root of a known number, meant…to find the value of one of the two equal dimensions of a square of a known area".[42] They used a technique which involved dissecting a square into various parts and making calculations regarding each section to determine the overall root.

The early method of calculating square roots from the *Nine Chapters* was known as the *Kai fan shu*, and it used Chinese counting rods and a counting board for the calculation. Historians generally believe this method was "essentially the same as that used by William Horner (1819) for solving higher numerical equations".[43] The method involved an algorithm which can be represented in modern notation in the following way:

> To take the square root of a $2k + 1$ or $2k + 2$ digit number N, the algorithm begins by finding the largest number $A_0 = a_0 \times 10^k$ where a_0 is a digit, such that $A_0^2 \leq N$. Then compute $N_1 = N - A_0^2$. Now find the largest $A_1 = a_1 \times 10^{k-1}$ such that $A_1(2A_0 + A_1) \leq N_1$, and form $N_2 = N_1 - A_1(2A_0 + A_1)$. Continue in this manner. If N is a perfect square, its square root will be the $(k + 1)$-digit number $S = a_0 a_1 \cdots a_k$.[44]

[41] Ibid.
[42] L. Wang & J. Needham, 'Horner's Method in Chinese Mathematics: Its Origins in the Root-Extraction Procedures of the Han Dynasty', p. 386.
[43] Lam Lay Yong, 'The Geometrical Basis of the Ancient Chinese Square-Root Method', p. 92
[44] Philip D. Straffin Jr, 'Liu Hui and the First Golden Age of Chinese Mathematics', p. 167

The traditional process begins with the number whose square root is to be extracted being depicted using counting rods on a counting board. In this instance, the number 71824, which Lam Lay Yong uses in his explanation of the process in ‘The Geometrical Basis of the Ancient Chinese Square-Root Method’, will be used. After the starting number is written down, which is traditionally called the *shih* or dividend, the lowest divisor, called the *hsia fa*, is placed under the number in the ones column. It is then shifted two places across, and then two places across again until it is under the ten thousands column. In this position, it “determines the hundredth place” of the root (see *figure 1*).[45]

7	1	8	2	4
1				

Figure 1 - The shih and hsia fa as they would appear on a counting board

In Yong’s translation we are next told that “from the *shih*, obtain the number for the first *shang”*, with the *shang* being a quotient.[46] The number of the first *shang* is 2, which we are told is “obtained by trial”. [47]

The first place of the root should be equal to or lower than the value of the *shih*. If 1 were to be used, which gives 100 due to the hundredth place being sought, 100 is multiplied by 100 to result in 10000. For 2, the value 200 is multiplied by 200 resulting in 40000. For 3, as before 300 is multiplied by 300 resulting in 90000. From

[45] Lay Yong, op. cit., p. 93
[46] Ibid.
[47] Lay Yong, op. cit., p. 98

these calculations, it can be seen that the ideal value is 200, and thus 2 should be used for the first place value of the square root. For as Lay Yong explains "if 300 is taken as a root, then it would be found that the product…300 (*shang*) x 300 (*shang*) exceeds the *shih*. Hence 300 is not a possible figure as the root, and the largest possible number is 200".[48]

One then multiplies the first *shang* (200) by the *hsia fa* and calls this the *fang fa*. The first *shang* (200) and the *fang fa* (200) are multiplied. The value this produces, 40000, is subtracted from the first *shih*, resulting in 31821 remaining. The *shang* and the *fang fa* are added to the board. After this, 2 is placed over top of the *shih* (see *figure 2*).

		2		
3	1	8	2	4
4				
1				

Figure 2 - the first shang, the shih, the fang fa, and the hsia fa

Next, the *fang fa* is shifted over one place and now given the name *lien*. The *hsia fa* is shifted two places over to the hundreds column. The process then begins to be repeated again. The value for the second *shang* is obtained by trial and error, and the value used this time is 6. Beside the *lien*, the value of the second *shang* multiplied by the *hsia fa* is placed (60), and it is called the *yü*. The *lien* and the *yü* are then

[48] Ibid.

added, and result (460) multiplied by the second *shang.* The resulting value, 27600, is then subtracted from the *shih* leaving 4224 (*figure 3).*

	2	6	
4	2	2	4
4	6		
	1		

Figure 3 - After the calculation and subtraction of the second shang

The process is then repeated one last time. The next *shang* found by trial and error is 8. We add 60 to the value of the *lien* and the *yü* combined to get 520. The new *shang* is added to this, making it 528. This value is then multiplied by the new *shang* to produce 4224. When this is subtracted from the *shih*, there is no remainder and thus the root has been located (*figure 4*).

	2	6	8
4	2	2	4
	5	2	8
			1

Figure 4 - The board after the last round of calculation

Yoshida Mitsuyoshi is known to have had access to the *Suanfa Tongzong*, which as mentioned dealt with many mathematical subjects of *Nine Chapters* including square root extraction. The method found in the *Suanfa Tongzong* is attributed to Liu Hui劉徽, a Chinese mathematician who lived in the third century CE and produced a very popular commentary on the *Nine Chapters* text. He famously added to the *Nine Chapters* method, and provided geometrical representations that allowed for the method to be understood as visually dissecting a square (similar to *figure 5*). He associated regions to be dissected with different colours – azure, red, and yellow – which were standard in China and connected to the Chinese theory of five elements.[49]

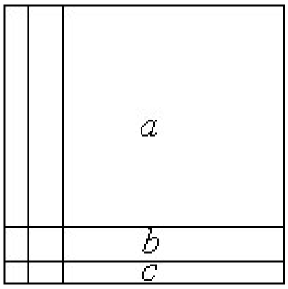

Figure 5

When geometrically representing the square and the parts to be dissected, Liu Hui also determined a new method of calculation which involved finding the value of the "small side…of the rectangles" used for the second place of the square root (represented by *b* in *figure 5*).[50] He was able to recognise that the total area of the two rectangles that made up this part of the square (*b* + *b*) was less than the area of the

[49] Jean-Claude Martzloff, *A History of Chinese Mathematics*, p. 223
[50] Ibid.

gnomon formed by combing the areas of this section, such that, where A is the square, x^2 is the largest possible integer found by trial and error for the first *shang*, and 2x is the *shang* times two:

$$y \leq \frac{A - x^2}{2x}$$

Liu Hui also worked with square roots that were not whole numbers. Robert Cohen explains his method:

> When the square root is not a whole number, then let $N = a^2 + r$
>
> ...the approximate value of the of the square root $\sqrt{N}$ is between
>
> $a + [\frac{r}{2a+1}]$ and $a + \left[\frac{1}{2a}\right]$.[51]

As well as this, Liu Hui also used a new approach to calculating roots which was to derive "minute numbers" to increase the preciseness of the value of the root and which would "constitute a decimal fraction".[52]

Now that Chinese square root extraction methods have been illustrated, how they impacted the method of Yoshida and thus his mathematics in the *Jinkōki* will be shown.

[51] Robert Sonné Cohen, *Chinese Studies in the history and philosophy of science and technology*, p. 247

[52] Cohen, op. cit., p. 247

Yoshida's Square Root Extraction Method

The square root extraction method of Yoshida Mitsuyoshi found in the *Jinkōki*, as will be shown, is clearly derived from the Chinese methods examined above. However, as mentioned, Yoshida does not make an outright copy, and alters the methods to bring a uniquely Japanese flavour to them. As well as this, he also adopts different tools for calculating the root.

In part III of the *Jinkōki* Yoshida's method for calculating square roots is found. The process begins first by Yoshida presenting a word question with a feel of practicality that is however only cosmetic. It reads: *if there is a square field whose area is 15,129 then what is the length of each side?*[53]

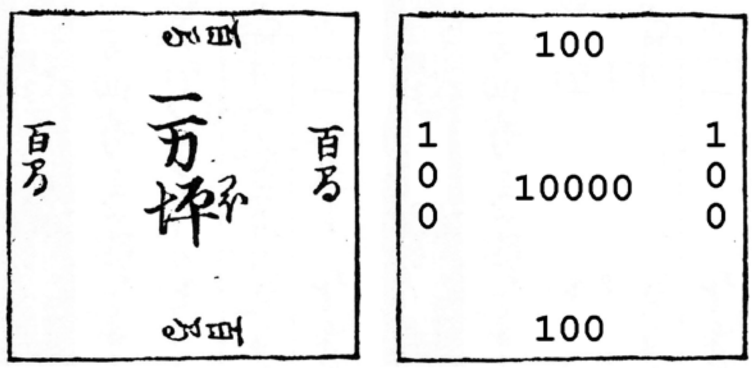

Image 3 – The original square and the translation

The finding of the solution starts with a square being drawn (*image 3*) which has a side length of 100 and a total area of 10,000. Why Yoshida chooses the value of 100 for the square's side lengths is not entirely clear, and no supplementary information is provided regarding this. It is likely however that he possibly wanted to

[53] Though it is put in a form which would make it useful for farmers, it is the case that historically farmers' fields have been rectangular in shape in Japan.

start from a multiple of ten that gave a total area size close to but below the starting area of 15,129. For the calculation process itself, Yoshida provides four drawings depicting a *soroban* (*image 4*) with places 10000–1000–100–10–1 which for explanation purposes will be referred to as **S-1**. In the first *soroban* drawing **S-1**, the value of the starting square's side length, 100, is entered. In the second, the area of the square – in this case 15,129 – is placed. In the third, the product of the first drawings value times itself is entered, being 10,000. Lastly, in the fourth drawing of **S-1** the value 100 is put again.

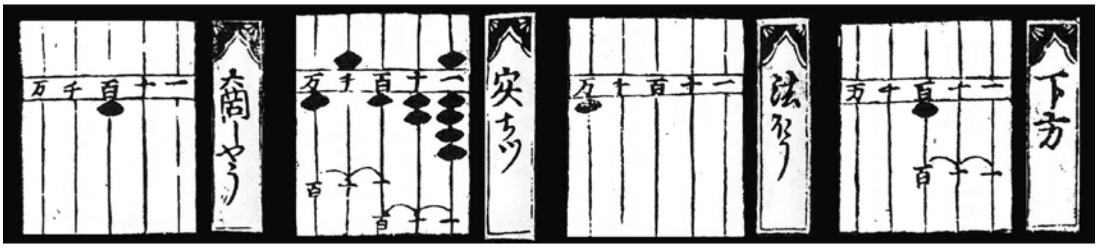

Image 4 –**S-1** as shown in the *Jinkōki*

Following this, the value from the third drawing of **S-1** is subtracted from the second, and a new set of four drawings (*image 5*) which we will call **S-2** appears.

The value of 5,129 derived from the prior calculation is placed in the second *soroban* drawing of **S-2**, and the value 20 is added to the first drawing. Why the value 20 in particular is chosen and added is again uncertain. Takenouchi however remarks on why it may be being used:

> How to get this 20 is a question. No explanation is made about this, but we consider this is done as follows. One will try 10, 20, 30, ….,

and do the process given below. Then, as the product, one gets 2,100, 4,400, 6,900, …, and one will choose the case so that the product is smaller than the 5, 129, and the largest amongst them.[54]

This method of ‘trial and error’ is something previously seen in the Chinese methods examined, and it is very likely that an influence and transmission of knowledge occurred here.

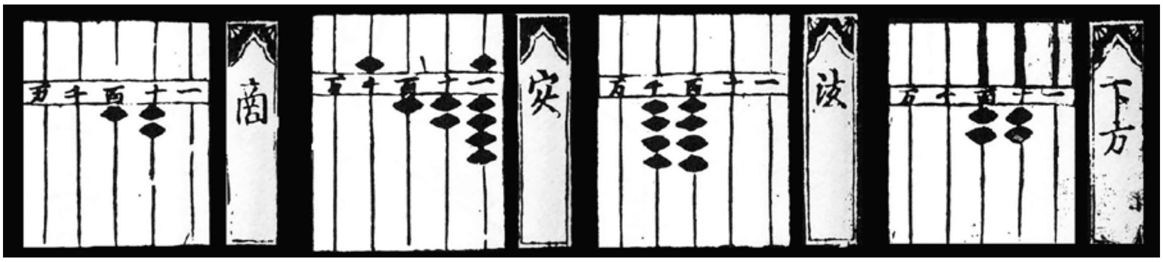

Image 5 – New set of *soroban* calculation boxes **S-2**

Coming back to the calculation, the value 120 is added to the third image - giving 220 – and then put in the forth. This number is then multiplied by 20, yielding 4400, which is added to the third image. As before, the value in the third image is subtracted from that in the second and the result – 729 – added to the second image of a new set of boxes **S-3**.

The number 3 is added to the value in image one of **S-3**, making it 123, which is added to the first image. In the fourth image of **S-3** we add 3 and the number 20,

[54] Takenouchi, op. cit., p. 165

making it now 243. This number is multiplied by the new number added, 3, yielding 729 which is placed in the new image three.

Now when the number in image three is subtracted from image two we are left with zero, which means the number in image one is the correct answer and thus the square root initially sought.

The process can be symbolically represented in the following way. We assign *a*, *b*, *c* and *d* to represent the four soroban based calculation boxes, *n* the starting number whose square root we which to discover, and *w*, *x* and *y* represent values added with each cycle.

$a = x$	$a_1 = x + y$	$a_2 = a_1 + z$	$a_3 = a_2$
$b = n$	$b_1 = b - c$	$b_2 = b_1 - c^1$	$b_3 = b_2 - c_2$
$c = x^2$	$c_1 = d_1 \times y$	$c_2 = d_2 \times z$	$c_3 = c_2$
$d = x$	$d_1 = [x + (x + y)]$	$d_2 = [d_1 + (y + z)]$	$d_3 = d_2$

Now, the values given in the problem can be placed into this representation, with w = 100, n = 15129, x = 20, and y = 3

$a = 100$	$a_1 = 100 + 20$	$a_2 = (100 + 20) + 3$
$b = 15129$	$b_1 = 15129 - 10000$	$b_2 = 5129 - 4400$
$c = 10000$	$c_1 = [100 + (20 + 100)] \times 20$	$c_2 = [(100 + (20 + 100)) + (20 + 3)] \times 3$
$d = 100$	$d_1 = 100 + (20 + 100)$	$d_2 = [100 + (20 + 100)] + (20 + 3)$

$a_3 = (100 + 20) + 3 = 123$

$b_3 = 729 - 729 = 0$

$c_3 = [((100 + 20) + 100) + (20 + 2)] \times 3 = 729$

$d_3 = [(100 + 20) + 100] + (20 + 2) = 243$

Unfortunately Yoshida does not provide any detailed information regarding what each step is meant to accomplish and why the boxes function the way they do. But, understanding can be found in hindsight.

In *a* appears the side length of a square whose initial area is a value below (yet close to) the actual area we wish to find the square root of. In each cycle an additional value is added to *a,* which is added to the original starting value and increases the side length of the square we started out with. This process repeats until *a* eventually contains the correct side length for the square whose area we originally set out to find.

In *b* we initially find the area of the square whose square root (and side length) we wish to eventually uncover. In the first cycle the 'estimate' area which is the result of the value added to *a* is subtracted from the actual area that was placed originally in *b*. What is going on here is this, by subtracting the estimated area from the actual area we are left with a 'margin of error'. Using *c* and *d* a new estimate area which is close to but below this 'margin of error' area is calculated and in the next cycle removed, leaving a new but decreased 'margin of error'. When we eventually get to the stage in which the 'margin of error' becomes zero we know that the correct square root has been found.

The value of *c* is the result of calculation in *d*. In *d* we find the sum of the area left after we increase the side length of the 'estimate' square and have subtracted the area that the 'estimate' square had before we added to its side length.

Breaking down the second image, the area values are (100 × 20) + (100 × 20) + (20 × 20). If we rearrange these values, removing the '× 20' common to each, we have (100 + 100 + 20). This is the value that is desired for d_1, but because we already have 100 in there we only add 120 to *d*. We then bring back the removed part, and times the equation by 20 and place this value in c_1.

This cycle repeats again as we add to a. After removing 4400 from the 'margin of error' we are left with a new area. Breaking down the parts left, as illustrated, we have [(100 + 20) × 3] + [(100 + 20) × 3] + (3 × 3).

Removing '× 3' we are left with [(100 + 20) + (100 + 20) + 3]. We already have 220 in d_1 so we add 23 and then place this value in d_2, times it by the removed 3, and put the value in c_2. When c_2 is removed from b_2 the result is zero, meaning we no longer have a 'margin of error' to deal with, and have found the correct side length in *a*.

The explanations provided may be erroneous, and there is no way of knowing with certainty what Yoshida believed *a*, *b*, *c* and *d* to be accomplishing. However, these explanations fit reasonably well with the figures and diagrams given.

Connection with Chinese Methods

We can see that there is a significant correlation between the Chinese square root methods previously examined and that of Yoshida. In his method, Yoshida can be seen to use 'trial and error' and the dissecting of a square procedure. However, he uses the *soroban* rather than counting rods and a counting board to calculate it and does not assign colours to each section like Liu Hui. Some of his calculations also differ, and he does not appear to implement some elements like the *hsia fa*. However, the multiplication of the *shang* and *fang fa* and subtraction of their product from the *shih* closely resembles the multiplication of *a* and *d* and subsequent subtraction of their product from *b* in the first stage of his square root extraction process. In Yoshida's method no explanation is given as to why he picks the number for *a* that he did, but the number being subtracted from *b* is as with the Chinese case a number close to but below the *shih/b*, meaning that while he does not explicitly state he uses this trial and error method it is extremely likely this is how he did obtain the values of 100, 20, and 3 in his square root calculation.

We also see similarity to the method of Liu Hui in that the square is essentially dissected into squares and gnomons, and the values of the area of these sections are slowly subtracted away. Liu Hui's geometrical diagrams are very nearly the same as those we find in Yoshida's method, although as stated Yoshida did not add colours to different sections which were dissected away. Yoshida also does not seek precision of the square root value by finding 'minute numbers' as Liu Hui did, and only dealt with whole numbers.

We can see here however a transmission of knowledge from China to Japan occurring, and also a converting of Chinese square root methods into something more uniquely Japanese. This is because Yoshida's method did not use the colour scheme of Liu Hui which had deep significance for the Chinese, and he adopted the *soroban* rather than Chinese counting rods for all calculations. This method also left out much information regarding how to calculate values like the shang/*a*, and used Japanese terms for values, such as *shou* instead of *shang*.

This tells us that Yoshida was influenced by Chinese mathematics, and indicates the importance of the study and circulation of certain Chinese works prior to the Edo period. Thus the square root extraction method of the *Jinkōki* was dependent upon this prior tradition.

Supra-utilitarian Elements

The square root extraction method of Yoshida, while being influenced by Chinese methods and thus context-dependent, can also however be understood as supra-utilitarian due to its very lack of influence from other contextual factors. His square root extraction method pulls away from the utilitarian and commercial themes of the *Jinkōki,* for while it is framed such that instructions are provided for finding the side length of any square field, the problem can be considered pseudo-practical because Japanese farmers did not traditionally use square fields.

In around 645 CE, the *jori* system village system was introduced in Japan.[55] This system saw villages "laid out on a rectangular or grid pattern under a system of land division" and many "fields in the Kyoto-Nara-Osaka region retain the dimensions of the ancient jori system" in Japan to this day.[56] As well as *jori* villages, *shinden* village types also appeared in the Edo period. These were settlements where land had been reclaimed and also saw fields largely take on rectangular shapes (although with both village types there were occasionally fields of other sizes depending on the landscape).[57] This indicates that rectangular fields were more preferable and prevalent in Japan than square fields. This being the case, needing to calculate the side length of square fields would have been a rare (though still plausible) task.

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[55] Pradyumna P. Karan, *Japan In the 21st Century: Environment, Economy, and Society*, pp. 205
[56] Karan, op. cit., p. 206
[57] Ibid.

Also, it seems strange that a farmer would start out first with the area of a field and then try to determine from this area the length of each side of the field. This is because without knowing the length of at least one side of the field to start with a farmer would not actually be able to calculate its area. Given these factors, the square root extraction method in particular does appear to be in fact pseudo-practical and supra-utilitarian, and indicates that the inclusion of Chinese inspired mathematics may have been purposeful and for specific reasons differing to those previously discussed. These reasons may have included a desire to teach more advanced mathematics to those who wanted to go beyond the basics. It may have also been a means to establish a distinction between everyday commercial mathematics and mathematics of a more supra-utilitarian and abstract nature.

Difficult Problems and *Idai*

Other examples of supra-utilitarian mathematics inspired by Chinese work can be seen in the *idai* problems included in the November 1641 CE edition of the *Jinkōki*.

In the *Suanfa Tongzong,* shown already have influenced the square root extraction method of Yoshida, there was a collection of 'difficult problems' included which were:

> …classified according to the ancient categories of the Nine Chapters and compiled in a pleasing, but intentionally disconcerting and surprising way, even for specialists in computation…they appeared difficult…They were intended to show off the virtuosity and professional

skill of arithmeticians in comparison with the mass of those with no arithmetical competence.[58]

In the *Jinkōki* there is also a selection of more difficult problems left up to the reader to solve which were included only in and after the November 1641 CE edition. These problems were known as *idai* and did not include instructions for their solution or an answer, and were often of a more supra-utilitarian nature.

Commentators such as Jean-Claude Martzloff believe *idai* may have been influenced by this section of 'difficult problems' in the *Suanfa Tongzong*.[59] Martzloff thinks in particular the 'cutting of a circle' *idai* problem was "perhaps inspired by the following found in the Suanfa tongzong: 'A small river cuts right across a circular field whose area is unknown; (b) given the diameter of the field and the breadth of the river find the area of the non-flooded part of the field'".[60]

The 'cutting a circle' problem which appears as a supplement to the November 1641 CE version of the *Jinkōki* concerns finding chord (line segments whose ends touch the circumference) and sagitta lengths in a circle that has been divided.[61] It presents as follows:

There is a circle-shaped area with a diameter 100 ken. A man wants to share it to three persons.

[58] Martzloff, op. cit., p. 56
[59] Martzloff, op. cit., p. 163
[60] Martzloff, op. cit., p. 162
[61] Rothman and Hidetoshi explain that the 'sagitta' is a "trigonometric function unfortunately not much in use nowadays…It is also known as the versine". Rothman and Hidetoshi, *Sacred Mathematics: Japanese Temple Geometry,* p. 304.

- The first one will have 2900 tsubo from the north.

- The second will have 2500 tsubo.

- The third one will have 2500 tsubo.

Then, how long will be the length of the sagitta from the north and the length of the chord? And, how long will be the length of the sagitta and the length of the chord in the middle?

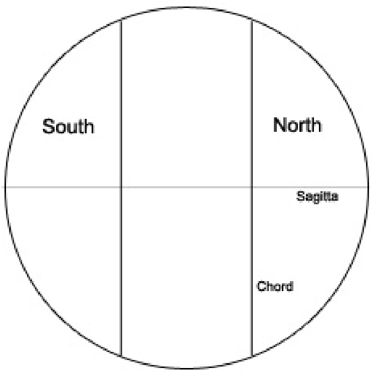

Figure 6 - Visual representation of 'cutting a circle' problem

This particular problem noticeably comes across as more of a challenge or puzzle than those previously examined from the text. It is harder than other problems, has no instructions or answer, is of a more recreational nature, and does have some similarity to the problem discussed by Martzloff. Given the influence the *Suanfa Tongzong* had on other areas of Yoshida's work (such as square root extraction) the likelihood of this problem being influenced by that found in the Chinese text is quite high, although it has obviously been much altered by Yoshida to make it more original.

Given this, *idai* problems can be seen as another way in which the *Jinkōki* was dependent upon the Chinese mathematical tradition for its content. This reliance on Chinese mathematics is another way in which the mathematics of Yoshida in general was dependent upon contextual factors. As well as this, *idai* further example supra-utilitarian mathematics in the text, and show that the purpose and scope of the *Jinkōki* changed over time. This is because before their inclusion the work was more orientated towards useful, utilitarian mathematics (for even the less utilitarian square root extraction method was instructional). The reason for this change and the inclusion of *idai* was related to public reaction towards the earlier versions of the text, which will be discussed in more detail later in this chapter.

1.5. National Seclusion, Nationalism, Adaption

Some interconnected contextual influences which had an impact on the form and content of the mathematics of Yoshida Mitsuyoshi in the *Jinkōki* include the national seclusion policy, the nationalist and anti-foreign attitudes of the *shōgunate*, and the history of adapting Chinese culture in Japan. It will be shown in this section how the alterations Yoshida made to the Chinese square root extraction methods were partly the result of factors such as these, and also caused him to write the *Jinkōki* in everyday Japanese.

Alteration and Adaption from Chinese

The adoption of Chinese culture and knowledge and adaption of it into something uniquely Japanese has been a common practice since the Yayoi period

(300 BCE – 250 CE). For example, in the mid-sixth century, Buddhism found its way to Japan from China, but instead becoming the new dominating religion in Japan it "settled into an easy complementary relationship with Shinto" – the indigenous belief system of the country – creating a harmonious, uniquely Japanese mix of Shinto and Buddhism that still exists today in Japanese society.[62]

Like Buddhism, mathematics was also transmitted to Japan from China. Around the year 284 CE, Chinese ideograms made their way to Japan and inspired the Sino-Japanese number system still used today.[63]

Chinese counting rods are also another example of how the Japanese adapted Chinese tools and knowledge. Chinese bamboo counting rods are believed to have made their way to Japan from China "as early as 600 A.D.".[64] These rods, known to the Japanese as *sangi*, were traditionally round cylinders but because of their tendency to roll and shift the Japanese altered them to be square.[65] These Japanese style counting rods were used exclusively until the time of the Edo period for calculation, at which point the adapted form of the Chinese *suanpan* abacus – the *soroban* – also became popular.

A further example of adaptation and alteration can also be seen in the square root extraction method of Yoshida. As discussed, the mathematical method appears to have been derived from the Chinese (particularly the square root extraction method of the *Suanfa Tongzong*), but it did have unique differences. These differences may have been inspired by this general trend of adoption and alteration that occurred in Japan. However, it is the case that prior to the Edo period no mathematical works had been

[62] Karan, op. cit., p.53
[63] Smith and Mikami, op. cit., p. 2
[64] Smith and Mikami, op. cit., p. 47
[65] Smith and Mikami, op. cit., p. 23

altered and it was acceptable to copy and study pure Chinese works. This means additional factors were at play in Yoshida's alteration of Chinese mathematics in the *Jinkōki*, and one of these was the isolation policy of the Tokugawa *shōgunate* and their increasingly anti-foreign attitude.

Isolation Inspiration

In the year 1630 CE, one year after the first edition of the *Jinkōki* was published, the *shōgun* Iemitsu Tokugawa徳川 家光banned the reading and selling of many foreign books with the edict of Kanei. In 1639 CE, he then enforced *sakoku* – the national seclusion policy – which saw all foreigners banned from Japan other than select Chinese and Dutch traders at Nagasaki port.[66]

The edict of Kanei saw books banned in two areas, the first being "religious and miscellaneous, the second scientific".[67] While the ban was largely to stop the spread of Christianity, and books containing Christian themes were targeted, among the banned books included treatises such as *Principles of Geometry, Practical Arithmetic, A Short Treatise on Geometry, A Treatise on the Theory of Rectangle-triangle.*[68] Shio Sakanishi writes that on one particular list of banned books uncovered, "out of twenty titles…only seven are on Christianity; the remaining thirteen are scientific works on subjects such as mathematics, astronomy, and geography".[69] This was because officials believed "the teaching of science was often made a cloak for Christian proselytising, and the government made little distinction

[66] Shio Sakanishi, 'Prohibition of Import of Certain Chinese Books and the Policy of the Edo Government', p. 290
[67] Sakanishi, op. cit., p. 292
[68] Sakanishi, op. cit., p. 293
[69] Sakanishi, op. cit., p. 294

between the purely evangelical and scientific works".[70] Because of this, many foreign works on science and mathematics were targeted by the government and the ban, whether they contained Christian themes or not.

Most mathematical works circulating in Japan prior to this time were Chinese, but the negative attitude towards foreign books (which, as has been shown, did specifically target scientific works as well as Christian) – combined with the general public's inability to read Chinese – meant that during this period when there was sharp economic growth it was difficult for citizens to access mathematical texts. Therefore, one of the reasons why alterations occurred in Yoshida's work is because of the isolation period and banning of foreign scientific works by the *shōgunate* happening to coincide with the time in which he lived and wrote.

It is the case as mentioned that Japan became a closed country during the time Yoshida lived, though not officially until 1639 CE. The banning on foreign works did also not occur until 1630 CE. However, anti-foreign sentiments and the process of closing Japan had already started prior to this, as evidenced by the executing and expelling of foreigners between 1622 and 1629 CE.

China was also not immune to the anti-foreign policies. Between the years 1592 and 1597 CE the Japanese invaded Korea twice, though their conquests were a failure partly due to involvement from the Chinese. Because of this, there was some strain in relations between Japan, Korea, and China during the start of the Edo period, and as well as a lack of Western foreigners in the country "Visitors from China during the Edo period…were extremely rare".[71]

[70] Ibid.
[71] Indra A. Levy, *Translation in Modern Japan,* p. 17

It was during this time of increasing anti-foreign attitudes and a revival of nationalist sentiments that Yoshida produced his mathematical work. Because of this atmosphere he may have found it more appropriate to provide useful mathematics from Chinese sources in a style more uniquely Japanese.[72] With foreign books on the verge of being banned at the time he wrote the *Jinkōki* – as the first version was published only four years before the ban was enforced – and many foreigners being executed and expelled during the very time it was published, he may have sought to minimise the foreign source of his work. There also could have been some risk of his work itself being banned were it to contain direct copies of Chinese methods.

Because the changes made by Yoshida to the Chinese methods were technically unnecessary – for the methods from China do work and can be understood to even be a little more advanced – and the fact that prior to the Edo period mathematicians seemed to have no problem in copying and teaching Chinese works, the changes made by Yoshida to mathematics such as the square root extraction method and *idai* problems were cosmetic and done on purpose. They were also likely designed to make the mathematics more unique to himself.

Therefore, the isolationist and nationalist sentiments of the *shōgunate* during the beginning of the Edo period likely influenced the adopting but altering of Chinese methods such as the square root extraction method and *idai* problems of Yoshida. This means the isolation period influenced the format of Yoshida's work, and his square root extraction method and *idai* problems were likely responsive to this political environment.

[72] Ju Brown and John Brown, *China, Japan, Korea: Culture and Customs,* p. 88

Impact on Language

While the isolation period, banning of books, and nationalist sentiments had an impact on the alteration of mathematics from Chinese sources in the *Jinkōki,* they also influenced the language of the text. Instead of being written in Chinese or *kanbun* 漢文 (an academic language which used Chinese characters and grammar but Japanese meanings and was commonly used by *samurai*, scholars, and many later mathematicians) the *Jinkōki* was published in everyday Japanese. This was not the traditional language for mathematical texts, and would not become fashionable among later mathematicians making this unique to Yoshida's mathematics.

The writing of this work in Japanese was not due to an inability on Yoshida's part to read Chinese, for he was influenced by Chinese mathematics (in particular Cheng Dawei's *Suanfa Tongzong*).[73] Given this, Yoshida would have had no problem writing a text in Chinese or *kanbun*. The fact he did not use this language, and instead used that of the lower classes, tells us his work was specifically designed to be used and read by average Japanese citizens.

The negative attitude towards foreign works (and their subsequent banning) combined with the need of the average people for mathematical training can be thus seen as influencing of the publishing of Yoshida's work in everyday Japanese.

1.6. Popularity and Status

Another way in which the mathematics of Yoshida Mitsuyoshi can be understood to be dependent upon and shaped by context is due to its own success and

[73] Chinese was the language mathematical books were written in during this time, for it was not until Yoshida published his work that mathematics appeared in the native Japanese language.

the impact it had on the Japanese public. Due to the popularity of the early editions of the *Jinkōki,* the November 1641 CE edition had presentational changes and included a new section of difficult unsolved problems – the *idai.* These changes and the inclusion of new unsolved mathematics was partly a response to the plagiarisms appearing of his work as well as rivals texts.

Also, the inclusion of other somewhat non-utilitarian problems such as the square root extraction method may have been influenced by status and class issues, and potentially aimed at *samurai* who were made up a large percentage of amateur mathematicians and were not allowed to trade during the Edo period.

Plagiarism

Yoshida Mitsuyoshi's *Jinkōki* was undoubtedly one of the most published works of the Edo period, with "several thousand copies…printed in the Kan'ei era (1624-1644) alone".[74] It was also one of the most plagiarised, with David Swain and Masayoshi Sugimoto writing that plagiarised versions of the *Jinkōki* numbered "more than four hundred".[75]

Idai problems were included due to the popularity of the early versions of the text. Through them Yoshida responded to the plagiarisms of his work which were appearing in great numbers and were often inaccurate. In the preface to the second book in the November 1641 CE edition of the *Jinkōki,* where *idai* problems first appear, Yoshida writes:

[74] Sugimoto and Swain, op. cit., p. 206, footnotes
[75] Ibid.

> Here in this book, in the place of division, I marked each step of the division process and multiplication by divisors in white-on-black. This is to make people know the right method of processing. However, there are men, who with poor knowledge of calculation, publish books imitating mine, but mark all the numbers in black in their books. This is a big fault. But general people do not recognize that failure. I know publish a new edition in the right form for the sake of the public.[76]

In this passage, we can see how to combat bad plagiarisms of his text Yoshida alters the way he marks numbers in the November 1641 CE edition. He also writes the following in the preface for book III:

> Some persons are known to be very able in making calculations. But, without entering deeply into this domain of learning, one cannot well establish their abilities….The general public will not be able to distinguish the person to be respected as the teacher of mathematics. I here present problems without giving the answers. From now on, a man who intends to teach or publish a book of mathematics should find the way by himself to answer these problems.[77]

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[76] Takenouchi, op. cit., p. 175
[77] Ibid.

From these two passages, we can see that Yoshida wanted to deter plagiarism of his work and to make sure that the mathematical texts which people were reading and relying upon were providing correct methods of calculation. He states explicitly in the second passage that those wishing to publish books on mathematics should first be able to solve the *idai* problems he has included, which is a strong indication that these problems were there as a means to combat inaccurate plagiarisms.

Also, it is the case as discussed that *idai* problems seem heavily influenced by problems in the *Suanfa Tongzong.* However, Yoshida had access to this book and its mathematics when he published the first editions of the *Jinkōki.* Even though this was so, he did not include *idai* problems similar to the 'difficult problems' of the *Suanfa Tongzong* until the November 1641 CE when his work had become popular, plagiarised, and rivalled. Therefore, these problems were context-dependent, for they were included as a response to actions and activities occurring in the Edo period environment.

Competition and Status

As well as combating plagiarised copies of his work, Yoshida's inclusion of *idai* in the November 1641 edition of the *Jinkōki* also served a competitive purpose.

Shigeru Jochi believes the appearance of other rival mathematical books published after the first edition of the *Jinkōki* "competed with Yoshida's, so he devised a new method, the *idai*…in his 1641 edition" in what could have been an attempt to give his work a fresh, new element that could not easily be copied and generated by other mathematicians.[78] In including these problems, Yoshida may have been attempting to create a distinction between his work and that of rival mathematicians.

But as well as this, the form of these problems also gives them a unique recreational and challenging nature. Jens Høyrup explains that in past mathematical cultures, recreational mathematics is sometimes believed to serve "as a means to *display virtuosity*, and…to demonstrate the status of the profession as a whole as consisting of expert specialist, and, on the other hand, to let the single members of the profession stand out".[79] Taking this into account, Yoshida may have also included these problems not only to make his work stand out from others but to elevate his own status as a mathematician above his rivals. But as well as this, including *idai* problems was a way for aspiring mathematicians to also test their abilities, and allowed the reader themselves to increase their status as a talented mathematician by solving and understanding them.

Idai problems may have been also included to create or display a distinction between practical, utilitarian mathematics done by merchants, artisans, and farmers and more advanced, theoretical supra-utilitarian work likely to be preferred by those in the *samurai* class. During the early part of the Edo period when the *Jinkōki* was published, "most mathematicians came from the samurai population", with roughly "75 percent of mathematicians" being *samurai*.[80] The *Jinkōki* however was popular with the lower merchant, artisan, and farming classes, so much that its "extensive use fixed the public image of mathematics as a tool for utilitarian ends".[81]

[78] Jochi (2000), op. cit., p. 428
[79] Jens Høyrup, *Lengths, widths, surfaces: a portrait of old Babylonian algebra and its kin*, p. 365
[80] James R. Bartholomew, *The Formation of Science in Japan*, pp. 18-9
[81] Sugimoto and Swain, op. cit., p. 207

Because "Samurai were prohibited legally from engaging in trade", and the Neo-Confucian class system promoted the separation of each class, *idai* problems would have been more favourable to *samurai* than other mathematics in the text.[82] It is also likely that many of the rival mathematical works were written by *samurai* given they made up such a large percentage of those practicing mathematics during this time. This may further suggest that these problems had an element of class and status distinction to them.

Another aspect Yoshida's mathematics additional to *idai* which may have been influenced by status and class during the Edo period is the square root extraction method. While it is presented in an agriculturally applicable manner as discussed, this seems only cosmetic and the problem in fact pseudo-practical.

The square root extraction method actually shows up in book III of the *Jinkōki,* which Takenouchi feels "contains problems which may interest amateurs in mathematics".[83] Given the nature of the square root extraction method, it seems this is a good interpretation, and it tells us that Yoshida was providing for those who had an interest in mathematics and wanted to go beyond commercial and agriculturally applicable problems.

For these reasons, Yoshida's *idai* and pseudo-practical problems were context-dependent not only in their dependence upon Chinese mathematics but also as a response to plagiarisms and rival works that appeared. As well as this, it may be the case that they were a way for amateur and professional mathematicians – as well as Yoshida himself – to prove their mathematical abilities and elevate their own status

[82] Charles D. Sheldon, 'Merchants and Society in Tokugawa Japan', p. 479
[83] Takenouchi, op. cit., p. 12

and that of the discipline itself as something studied for its own sake.

Legacy

Lastly, the impact that the inclusion of *idai* problems had on Japanese society and how they themselves became influential on other mathematics of the period will be briefly mentioned.

The inclusion of *idai* in mathematical textbooks became a popular tradition in the Edo period after it was first done by Yoshida. The activity was “copied by others and led to ongoing mathematical developments in Japan”.[84] It became the case that whenever later “wasan mathematicians published a book, they proposed unsolved problems at the end”.[85] Also, as we will see later, *idai* may have influenced the development and nature of later mathematics such as the *sangaku* tradition.

The survival of so many copies this textbook in the modern day indicates that the work itself had a big impact on Edo period society. Due to its adequate responding to Edo period context and dependence upon it the text itself became something upon which later mathematics was dependent.

1.7. Summary

In this chapter, the ways in which Yoshida Mitsuyoshi’s mathematics of the *Jinkōki* was shaped by or dependent upon context was shown. Many factors such as the economic growth that occurred at the beginning of the Edo period, the enforcing of the alternative attendance policy, the isolation policy, the introduction of the

[84] William E. Deal, *Handbook to life in Medieval and Early Modern Japan*, p. 239
[85] Hiroshi Okumura, ‘Japanese Mathematics’, *Symmetry: Culture and Science*, Vol 12., Nos. 1-2, p. 79

soroban, the complicated currency system and economy based on around rice, the prior study and distribution of Chinese mathematical works in Japan, and even the popularity of his own work can all be seen to sculpt and influence the presentation, content, and language of this text.

The influences acting upon the *Jinkōki* differ to that of later mathematics however, evidencing how different contextual factors were important for different practitioners and why the mathematics of this period manifested in such different ways. The differing mathematics of Takebe Katahiro will now be examined, followed by that of the *sangaku* tradition.

CHAPTER 2

EDO PERIOD CONTEXT AND TAKEBE KATAHIRO'S MATHEMATICS

INTRODUCTION

The mathematics of Takebe Katahiro 建部 賢弘, who lived in Edo from 1664 to 1739 CE, differed in many ways to that of Yoshida Mitsuyoshi. In this chapter, Takebe's *Tetsujutsu Sankei* 綴術算経 (published in 1722 CE) will be examined and these differences explored. This will be done through an investigation of the ways in which this work was dependent upon context. It will be seen how many of the factors impacting this work did not impact Yoshida's, indicating how different factors were important for different practitioners in the Edo period and impacted mathematics in varying ways.

One evident difference for example between Takebe and Yoshida's work caused by contextual factors is the *Tetsujutsu Sankei*'s lack of commercial and agricultural mathematics (such as currency conversions and calculations relating to rice). Takebe's text dealt instead with abstract supra-utilitarian mathematics disconnected from economic context due to it being specifically written for an audience who were not farmers or merchants but rather the country's elite. The mathematics of the text was not framed in an instructional way either and did not present mathematics for specific commercial or agricultural situations. It was also of a complex nature which would not make it easily applicable to everyday situations.

For example, the text contained an algorithm which produced one of the most accurate approximations of π attained in the Japanese tradition.[86] While Yoshida provided an approximation of π in the *Jinkōki,* Takebe's approximation was of a level of precision that was beyond the point of actual practical usefulness, being correct to forty-one places. In the text Takebe also authored the first instance of infinite series and power series expansion in the Japanese tradition with his method for calculating $(\arcsin \theta)^2$. As this illustrates, the content of Takebe's mathematics was less practical than the work of Yoshida Mitsuyoshi and was not specifically designed to be practically applied to commercial and agricultural situations.

As well as this, Takebe's mathematics was not presented in ordinary Japanese. Instead he wrote the *Tetsujutsu Sankei* in the *kanbun* language, meaning only those specially trained in reading Chinese characters (who were usually the elite, academics, or other mathematicians) could make sense of the text. He also used terminology that was not standard amongst other mathematicians of the time (including Takebe's peers and family), being "situated halfway between everyday language" of Neo-Confucians and "the technical language of a mathematician".[87] This indicates his work did not just differ from Yoshida's but also to some of his contemporaries. Through its use of *kanbun* it also was made inaccessible to the common classes, meaning it was not designed for use by non-specialists.

The *Tetsujutsu Sankei* also included discussions pertaining to methodology and approaches to mathematics (which Annick Horiuchi also describes as a meta-mathematical theory) inspired it seems directly from Neo-Confucian philosophy.[88]

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[86] See Shigeru Jochi (1997), 'Takebe Katahiro', *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures,* p. 2078
[87] Annick Horiuchi, *Japanese Mathematics in the Edo Period (1600-1868),* p. 272
[88] Ibid.

Although other mathematicians adapted principles from this belief system, Takebe purposefully used terms from it to describe and express his new methodology. Takebe was also the only mathematician to purposefully include any discussion on methodology in the *Tetsujutsu Sankei*, making it "perhaps the only book on the methodology and philosophy of mathematics in the history of *Wasan*".[89] Takebe's contemplation of mathematics however shares some similarities with strains of thought which have appeared in the Western tradition, although it does present differences as well which will be shown later.

It was also the case that the text was part of the eight *shōgun*'s personal library and personally sent to the *shōgun* to read. All of this indicates that the *Tetsujutsu Sankei* was highly specialised and individualistic in nature, and seems to have had a very different purpose to the work of Yoshida Mitsuyoshi.

The *Tetsujutsu Sankei* was published ninety-three years after the first edition of the *Jinkōki.* There were many changes which occurred in Japanese society during this time, such as the growth in popularity of Neo-Confucianism and the ease on the ban against foreign works. Because of this contextual elements which earlier works such as the *Jinkōki* were dependent upon were no longer necessarily influential and important. This meant the ways in which the works of later practitioners such as Takebe were shaped by context often differed to Yoshida.

In this chapter, the main contextual factors of influence that shaped Takebe's mathematics in the *Tetsujutsu Sankei* are examined. These largely relate to the arrival and popularity of Neo-Confucianism in Japan and his career as a *shōgunate samurai*. It will be shown how these factors influenced the content and presentation of the

[89] Murata, op. cit., p. 107

Tetsujutsu Sankei, evidencing the work as context-dependent but impacted by different factors to Yoshida.

2.1. Neo-Confucianism

The mathematics of Takebe Katahiro, as mentioned, was impacted by the Neo-Confucianism belief system which became popular in Japan around the end of the seventeenth century. Both Takebe's methodology of mathematics and choice of terminology can be seen to be greatly shaped by the introduction of this belief system. This influence was largely due to the popularity Neo-Confucianism found within the government *shōgunate* office and Takebe's employment by the government as a scientific advisor.

Without an understanding of the Neo-Confucian belief system the mathematical methodology of Takebe is not initially easy to understand. This is because many of the concepts and terms (such as the principle *ri*) which Takebe used to express the new methodology he presented in the *Tetsujutsu Sankei* were adopted and adapted directly from Neo-Confucianism.[90] Because of this, the external influence Neo-Confucianism had on his work illustrates one way in which context was important for the development of Takebe's mathematics in the *Tetsujutsu Sankei*. The influence of this belief system, for instance, was not visible in the writings of Yoshida Mitsuyoshi and also had no great connection to the later *sangaku* tradition. This indicates that different contextual factors did help shape the mathematics of different practitioners in varying ways during the Edo period.

[90] Horiuchi, op. cit., p. 272

Neo-Confucianism in Japan

It was first during the Yamato period (250 – 710 CE) that the Chinese Confucian belief system was adopted by Japanese leaders as a means to unify Japan and gain support for a single leadership through a claim to divine rule.[91] In the Edo period an altered form of the belief system largely based off the writings of the Chinese scholar Zhu Xi entered into Japan and became popular amongst the elite.[92] This new form became known as Neo-Confucianism.

While early Confucianism much consolidated the rule of previous clans, Neo-Confucianism fostered the re-introduction of Confucian ideals of social hierarchy and altered prevailing sensibilities concerning the virtues. The influence it had on "Tokugawa thought and culture was undeniably deep".[93] In 1630 CE what would become the first official "Confucian academy" opened in Japan, and by 1872 CE there were at least 277 of these academies whose "function was, through a study of Confucian classics, to cultivate the moral character of the…local ruling classes".[94] Neo-Confucianism and its teaching thus grew rapidly from the middle of the seventeenth century and remained strong until the end of the Edo period.

The Tokugawa *shōgunate* came to particularly favour this belief system. They sought their "educational creed in Neo-Confucianism" and played a key role in its diffusion and acceptance in Japanese society.[95] The fifth *shōgun* Tokugawa Tsunayoshi 徳川 綱吉 (in office from 1680-1709 CE) was for instance highly enthusiastic about Neo-Confucianism and built a "Paragon Hall…near the center of

[91] See Kenneth G. Henshall, *A History of Japan: From Stone Age to Superpower*, pp. 11-21
[92] Thomas P. Kasulis, 'Sushi, Science, and Spirituality: Modern Japanese Philosophy and Its Views of Western Science', p. 231
[93] Tetsuo Najita, 'Intellectual Change in Early Eighteenth Century Tokugawa Confucianism', p. 931
[94] Ibid.
[95] Ibid.

Edo, with all the splendour of a state shrine" dedicated to Neo-Confucian studies.[96] He was devout to the extent that he is said to have even personally given lectures on Confucian classics at the Hall's annual commemoration ceremony.[97]

While the *shōgun* adopted and embraced the belief system in Edo, the diffusion of Neo-Confucianism throughout the country itself was greatly aided by other members of the Tokugawa family. For instance, Hoshina Masayuki (1611-1672 CE) and Tokugawa Mitsukuni (1628-1701 CE) helped Neo-Confucianism find its way "into many comparatively isolated regions that could be penetrated only slowly and with difficulty" through their patronship and strategic placement in rural areas.[98] Because of this, the belief system became known to both the elite and common people from the early-mid Edo period onwards.

The Belief System

In the Neo-Confucian tradition, a core component is "the notion of investigating natural things to understand their laws or principles".[99] Regarding metaphysics, Neo-Confucianism posits the existence of two components of reality. These are the *ri* 理, which is described as a "rational principle" or pattern, and the *ki* 気 which is "the vital, transformative stuff of all that exists, including things that are solid, liquid, and gaseous".[100]

The 'rational principle' *ri* can be understood as "the rational and moral order of things generally, as well as that of each and every thing in terms of their

[96] Wm. Theodore de Bary, Carol Gluck, & Arthur E. Tiedemann, *Sources of Japanese Tradition*, p. 75
[97] De Bary, Gluck, & Tiedemann, op. cit., p. 75
[98] De Bary, Gluck, & Tiedemann, op. cit., p. 76
[99] Thomas P. Kasulis, op. cit., p. 231
[100] John Tucker, 'Japanese Confucian Philosophy', *The Stanford Encyclopedia of Philosophy Online*, http://plato.stanford.edu/entries/japanese-confucian/

particulars".[101] This concept was largely based off the Chinese philosopher Zhu Xi's understanding of traditional Confucianism, where:

> Zhu Xi likened principle in things to a seed of grain, each seed having its own particularity but also manifesting generic, organic elements of structure, growth pattern, direction, and functional use, whereby each partakes of both unity (commonality) and diversity.[102]

For Japanese Neo-Confucians, *ri* had the same function. It largely corresponded to the order of the components of things and objects and their order in nature, being considered an 'organising' principle in this respect.

In Neo-Confucian philosophy, human nature and morality are considered *ri*. They order and regulate humans individually as well as collectively as a species. However, there is a deeper association with morality in *ri* than in the English term 'principle'. There is a "normative, moral side of *ri* which makes it as much an ethical aspect of existence as it is a rational one. In the case of humanity, as well as virtually all of the ten-thousand things of the cosmos, *ri* is defined as morally good".[103] Therefore, the concept of *ri* has a heavier sense of intrinsic moral goodness than the English term 'principle'.

The study of principles was important in the Neo-Confucian tradition. For example, in the philosophy of Zhu Xi there existed a method of 'investigation of things' which was concerned with "the study of principles and also self-cultivation to

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[101] Ibid.
[102] De Bary, Gluck, & Tiedemann, op. cit., p. 40
[103] Tucker, op. cit.

bring one's conduct into conformity with the principles that should govern it".[104] It was considered important to study principles for self-cultivation purposes because:

> The principle of a thing is, as in Platonism, perfect, but in being clothed in material form the possibility of imperfection and thus evil arises. In order to…move towards the perfection that is in their unique principle, men…must pursue mental cultivation, endeavouring to understand principles, and thereby their summation by the 'investigation of things'.[105]

The study of principles helped combat the evil and imperfection which might arise from contact or exposure with the material world. This 'investigation of things' however was not so much "an empirical investigation of the material world as a leap of intuitive insight".[106] The process of understanding it however was an occurrence of the material world and often consisted of devoted immersion with a topic which sparked an intuitive insight or moment of enlightenment, for:

> It is a specific feature of neo-Confucian philosophy to represent acquisition of knowledge as corporal experience, in which the subject seeks to become immersed in his object till attaining a point of maturation where the principle (*li/ri*) unveils itself.[107]

This understanding of principles and the 'investigation of things', as well as not being an empirical investigation, was not generally extended "into the realms of

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[104] De Bary, Gluck, & Tiedemann, op. cit., p. 40
[105] David Miller and Janet Coleman, *The Blackwell Encyclopaedia of Political Thought,* p. 66
[106] Ibid.
[107] Horiuchi, op. cit., p. 272

what we would call natural or social science".[108] The 'things' of investigation tended to be related to morality and human affairs, much like traditional Confucianism. However, these Neo-Confucian ideas were to find their way into the realm of mathematics and played a key role in shaping the work of Takebe Katahiro.

For instance, Neo-Confucianism is visibly evident in the mathematical methodology which Takebe presents in the *Tetsujutsu Sankei.* In this text he utilises the concept *ri* 理 and provides a mathematical methodology that resembles the process found in the Neo-Confucian idea of insight through immersion (in which one becomes immersed in a subject until the *ri* is unveiled in a point of maturation).

Annick Horiuchi writes that the inclusion of *ri* in Takebe's writings "sheds new light on the role played by neo-Confucian philosophy in the development of mathematics during the Edo period".[109] This is because it shows that thoughts on mathematics were being connected with and thus influenced by this belief system during the early to mid eighteenth century. In particular, Takebe can be seen to adapt Neo-Confucianism for purposes not connected to moral philosophy or self-cultivation, evidencing his work was shaped by the belief system.

Methodology and Neo-Confucianism

In the *Tetsujutsu Sankei,* Takebe included a discussion on mathematical methodology that was unprecedented in Japan. He moved beyond the previous scope of the discipline and considered the nature of mathematics and best approaches to it. Its debt to Neo-Confucianism is clear because in this text Takebe used many Neo-Confucian derived terms (such as *ri*) to describe and form his mathematical

[108] De Bary, Gluck, & Tiedemann, op. cit., p. 41
[109] Horiuchi, op. cit., p. 272

methodology. For instance he states that his methodology consists of:

> [N]othing else but the act of examining and of seeking by accumulation (*tsuzurite*) until one attains an understanding of the principle (*ri*) of the procedures.[110]

In this paragraph the term *ri* 理 is seen connected with mathematical procedures. Takebe also applies something similar to the Neo-Confucian 'investigation of things' to his mathematics, but rather than studying principles for the purpose of self-cultivation he studied them to understand mathematical procedures. For Takebe, the way to study the principles of mathematical procedures seems to have been through a process involving the examination and accumulation of numerical data and calculations, which will be talked about more shortly.

In a later paragraph of the *Tetsujutsu Sankei*, Takebe references the Neo-Confucian inspired idea of *ri* again in his definition of what it is to do mathematics. This indicates that both his methodology and general conception of the nature of mathematical activity were shaped by Neo-Confucian philosophy:

> Mathematics consists in establishing rules, clarifying the principle of the procedures, and calculating numbers.[111]

As discussed earlier, the *ri* of Neo-Confucianism is often thought of as a 'organising' or 'rational' principle. While it is difficult to be certain in hindsight what Takebe had in mind when connecting *ri* with mathematics, the grasping or clarifying

[110] Horiuchi, op. cit., p. 262
[111] Horiuchi, op. cit., p. 268

of the *ri* of mathematical procedures may have been the process of coming to a realisation of how a mathematical method worked by understanding the particular elements or mathematical objects used in it and their relationship. Regardless of whether this is accurate, as Horiuchi writes “What is certain is that Takebe, in elaborating his meta-mathematical theory, was led to borrow a part of his terminology…from neo-Confucian philosophy”.[112] Thus the mathematical *ri* of procedures was connected to the *ri* of Neo-Confucianism, and evidences an influence from this belief system on Takebe’s mathematics.

It is the case that Takebe’s want for an understanding of the *ri* of procedures also resembled Zhu Xi’s seeking of conformity with principles that govern ‘things’, meaning Zhu Xi could “have been a source of inspiration in the conception of the *tetsujutsu*”.[113]

As well as determining mathematical procedures through ‘accumulation’ until one understands the *ri* of them, Takebe also promoted another concept which seems connected to Neo-Confucianism. In the *Tetsujutsu Sankei*, Takebe discusses a method which resembles the process of investigation and enlightenment through immersion, in which one becomes “immersed in his object till attaining a point of maturation”.[114] Takebe writes:

> When the study of one case does not permit one to attain the
> principle of the procedure, one must investigate a second case.
> If these two cases are not enough, one investigates a third.
> Even if the principle of the procedure is deeply buried, when

[112] Horiuchi, op. cit., p. 272
[113] Ibid.
[114] Ibid.

one investigates many cases, there always comes a moment of maturation, and there is no case where research does not succeed in the end.[115]

This passage suggests that if the principle of a procedure is not attained from studying one mathematical case one should 'accumulate' more and examine as many as necessary until it becomes known in an intuitive 'moment of maturation'. Horiuchi terms this 'Research by means of Numbers'. This examination of computations is sometimes called "the most important concept in traditional Japanese mathematics" and was original to the *Tetsujutsu Sankei*.[116]

Research by means of numbers seems to have involved producing series of calculations and drawing insights from them about the mathematics being examined. For example, in producing the formula (arcsin θ)² Takebe used research by means of numbers and "computed small natural numbers and then predicted infinite numbers".[117] Shigeru Jochi explains his method:

He computed the length of curve AB (hereafter *s*) using the diameter *d* and the length of straight line AB (h). Takebe set up *d* = 10 and h = 10^{-5}. Then letting the half point of the straight line AB be C_2, and the half point of curve AB be B_2, he computed the length of $AB_2(h_2)$. Then he computed the length of AB_4 as h_4, and continued to compute h_8, h_{16}, h_{32}, and h_{64}. Takebe computed h_∞ using a sort of infinite series…and obtained h_∞. Second, he indicated this

[115] Horiuchi, op. cit., p. 262
[116] Rothman, 'Japanese Temple Geometry', *Scientific American,* Vol. 278, Issue 5, pp. 86-7
[117] Jochi (1997), op. cit., p. 2077

value using h and d. The power 10^{-4} is h by d, and the approximate value of the coefficient is 1….The power of 10^{-10} is h^2, and the approximate value of the co-efficient is 1/3. Takebe continued to compute as above, and he set the series as $\left(\frac{s}{2}\right)^2 = A_0 + A_1 + A_2 + A_3 + A_4 + \ldots\ldots$ He then expanded…to the general series of a_n, which was $A_n = \frac{2n^2}{(n+1)(n+2)}$. Therefore Takebe obtained the formula $\left(\frac{s}{2}\right)^2 = 2\sum_{n=0}^{\infty}\left(\frac{(n! \times 2^n)^2}{(2n+2)!} \times \frac{h^{n+1}}{d^{n-1}}\right)$.[118]

Going by Takebe's statement that one should investigate many 'cases' or computations, this procedure can be understood as the result of computing, examining, and expanding a series of calculations pertaining to curves of arcs. Through this act, and "apparently…trial and error", Takebe seems to have determined that an infinite series existed and then proceeded to formulate a procedure for calculating it as seen above.[119] He came to an understanding of the nature of the mathematical object he was investigating by "relying only on numerical calculations" and computing many instances of them.[120] An idea of the procedure is also only come to after an examination of multiple numerical calculations has occurred.

The philosophy behind this method can be understood as somewhat similar to immersion as a means to attain *ri* in the Neo-Confucian tradition. Takebe's approach however is one where the principle (and enlightenment) is attained through computation and investigation. For Takebe, it is this process of immersion with

[118] Jochi (1997), op. cit., pp. 2077-8.
[119] Rothman and Hidetoshi, op. cit., p. 304
[120] Horiuchi, op. cit., p. 264

accumulations of numerical data that produces a ‘eureka’ moment of maturation and allows one to grasp the *ri* of the mathematical procedure.

For Takebe, the understanding of mathematical procedures through computation and a flash of insight was also partly dependent upon an individual mathematician’s “capacity for solving mathematical problems”, for one could not “recognize any universal and analyzable approach leading to the truth” of procedures.[121] This added an esoteric and personal aspect to the methodology, as well as making the comprehension of truth in mathematics an almost subjective activity.

This individualist approach was still relatively in line with Neo-Confucianism however. Neo-Confucians believe that “the highest level of truth could only be conveyed in a mind-to-mind transmission, one that typically transcended the use of discursive language” due to the inability of language to adequately express the concepts.[122] Because of this, the attaining of truth was a subjective, individualistic experience.

While this kind of mathematical methodology may seem unique or surprising, it is the case that similar ideas and thoughts connecting mathematical practice and insight have occurred in the history of Western mathematics and philosophy. Johann Carl Friedrich Gauss (1777 – 1855) for example is believed to have spoken of “a sudden flash of insight” occurring in the mathematical creative process.[123] Henri Poincaré (1854–1912 CE) also wrote of mathematical insight with regard to “appreciation of the aesthetics of mathematics”.[124] In the modern day, this ‘eureka’ moment is also recognised by academics and is talked about as:

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[121] Horiuchi, op. cit., p. 265
[122] Tucker, op. cit.
[123] Jacques Hadamard, *The Mathematician’s Mind: The Psychology of Invention in the Mathematical Field*, p. 15
[124] Mary Barnes, ‘’Magical’ Moments in Mathematics: Insights into the Process of Coming to Know’, *For the Learning of Mathematics*, p. 33

> Making a new discovery or finding a new connection…compared with a light switching on…finding a new path through unfamiliar terrain or seeing how to fit pieces into a jigsaw…belonging to a key stage in the creative process.[125]

As this indicates, Takebe's thoughts pertaining to methodology were not original to the Japanese tradition because he expressed ideas similar to Gauss and other academics. Takebe did however explicitly shape his mathematical methodology around invoking this 'moment of maturation' and using it to induce understanding pertaining to mathematical procedures. Takebe understands this as the clarifying or understanding of the *ri* of mathematical procedures and makes it a core element of his mathematical methodology. Takebe also saw mathematics and the search and attainment of truth as a somewhat personal and subjective experience which only certain practitioners may be able to achieve. But nonetheless, while Takebe's conception of a moment of insight or maturation was not necessarily original it was influenced by Neo-Confucianism. Thus his work can be seen as being dependent upon this belief system and context.

<u>Summary</u>

In this section some of the ways in which Takebe's mathematical methodology in the *Tetsujutsu Sankei* was shaped by Neo-Confucianism were shown. It is the case that Takebe adopted Neo-Confucian concepts such as the *ri* and used

[125] Ibid.

them to help “express his concept of mathematics”.[126] However rather than seeking moral insights he sought mathematical ones. He also seemed to adapt the ‘investigation of things’ of Zhu Xi for mathematical purposes and thus sought to do mathematics by immersing himself in computations until the *ri* of the procedures became clear.

Due to this Takebe’s mathematical writings in the *Tetsujutsu Sankei* can be seen to be context-dependent and illustrative of another way in which context expressed itself in Japanese mathematics of the Edo period. This also shows how different contextual elements were important for different practitioners, for Takebe’s approach and methods were influenced by different elements to those impacting Yoshida Mitsuyoshi (such as Neo-Confucianism). These contextual factors, once understood, help explain why the approach, content, and presentation of Takebe’s work was of such a different manner to Yoshida’s.

2.2. Shōgunate Influence

As shown, some of the *Tetsujutsu Sankei*’s language and content can be seen to be shaped by context due to Takebe’s use of Neo-Confucian concepts. But it is the case as mentioned earlier that Takebe was not alone in adopting certain elements of Neo-Confucianism (particularly *ri*) for mathematical purposes. However, as will be discussed in this section, the ways in which he expressed the Neo-Confucian inspired concepts he adopted in the *Tetsujutsu Sankei* did differ to other mathematicians (including his own brother), and caused his mathematics to be more noticeably Neo-

[126] Horiuchi, op. cit., p. 272

Confucian. His discussion of methodology was also an inclusion not seen in the work of other practitioners.

One of the reasons why Takebe specifically used Neo-Confucian "language to express his concepts of mathematics" was due to the increasing popularity of Neo-Confucianism discussed in the last section.[127] But as well as this his personal connection to the *shōgunate* – who were great promoters of the belief system and aided its growth in the country – also directly impacted the Neo-Confucian naturalising of his mathematical methodology. This was because, as shall be shown, his audience and sponsors were the Neo-Confucian government rather than professional mathematicians operating independently of the *shōgunate.*

In this section, Takebe's connection to the government and career in the *shōgunate* office are discussed and the influence these factors had on the content, audience, and presentation of his work the *Tetsujutsu Sankei* shown.

Takebe and the Shōgunate

While Yoshida Mitsuyoshi came from a wealthy merchant family, Takebe Katahiro was born into a prestigious *shōgunate samurai* clan. Due to this he worked for the government in Edo from as early as 1683 CE in a prestigious "hereditary post of private secretary of the *bakufu*" held by his clan.[128]

During his early years working for the *shōgunate* office, Takebe's scientific "research was relegated to the back burner".[129] When Tokugawa Ienobu徳川 家宣 became *shōgun* in 1704 CE Takebe was "sometimes encouraged…to make

[127] Ibid.
[128] Horiuchi, op. cit., p. 116
[129] Horiuchi, op. cit., p. 119

astronomical instruments", though his mathematical endeavours remained largely separate from his work for the *shōgunate*.[130]

When Takebe wrote the *Tetsujutsu Sankei* in 1722 CE Tokugawa Yoshimune 徳川 吉宗 had inherited the title of *shōgun* (occurring in 1716 CE). This change of *shōgun* had a significant impact on Takebe's research – particularly his mathematics. This was because Yoshimune was not hostile towards advancements in science or the study of foreign knowledge, and was in fact enthusiastic and supportive of both. For example, in 1720 CE Yoshimune eased the ban on foreign works previously enforced by Tokugawa Iemitsu in 1633 CE. He is also known to have had a particularly keen interest in science, and it is said that it was "the fond dream of Yoshimune (1684-1751) to issue a calendar" during his reign.[131]

Among Yoshimune's interests was also cartography. Two years after attaining the title *shōgun,* Yoshimune "gave the order to check the accuracy of the Genroku maps and to proceed to the realization of a global map of Japan".[132] This was a task that required men trained in mathematics and science. It was then that Yoshimune first made use of Takebe, whose skills had been previously underutilised during the reigns of the previous *shōguns*. Takebe proved to be particularly useful and influential in the development of the new maps, for after initial struggles the *shōgunate* handed the entire project over to him. He is thought to have worked on this task for around four years and completed it in 1719 CE.[133] The map was officially added to the library of the *shōgun* in 1728 CE.[134]

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[130] Horiuchi, op. cit., p. 118
[131] De Bary, Gluck, & Tiedemann, op. cit., p. 315
[132] Horiuchi, op. cit., p. 209
[133] Smith and Mikami, op. cit., p. 146
[134] Horiuchi, op. cit., p. 211

Takebe also did much work on calendrical science for Yoshimune, even though the calendar reform itself was to be declared a failure.[135] For example, Takebe aided in the translation of a Chinese work – the *Lisuan Quanshui*曆算全書 (Complete works on Calendrical Astronomy and Mathematics) – which largely dealt with Western astronomy. In the preface to the *Lisuan Quanshui*, written in 1733 CE by Takebe himself, he greatly praises the *shōgun*:

> …since the present *shōgun* has reigned, the sun of scientific progress has arisen and shines high in the sky. Its light has reached various domains and even calendrical science, of which he has captured all subtlety. He gave the order to go to Nagasaki to lift the ban on books…We can say that we thus have gained the benefit of a presentation of the Western calendar.[136]

Not only does this passage show the reverence and respect Takebe had for Yoshimune, but it indicates the significant impact Yoshimune's progressing to *shōgun* had on science in Edo Japan. It can also be seen how Takebe was indeed actively involved in the scientific projects of the *shōgun*.

As well as being interested in astronomy and cartography Yoshimune also had a keen interest in mathematics, as a special manuscript of Takebe's *Tetsujutsu Sankei* was sent directly to the leader.[137] We are told by Horiuchi that the *Tetsujutsu Sankei*

[135] Horiuchi, op. cit., pp. 227-31
[136] Horiuchi, op. cit., p. 226
[137] Jochi (1997), op. cit., p. 2078

in fact appears “to have been written for the shōgun personally” and “belonged to the bedside books of the sovereign after he retired”.[138] This indicates that Yoshimune was indeed greatly interested in mathematics, and his keeping of the text into his retirement shows a particular fondness for the particularly heavily Neo-Confucian *Tetsujutsu Sankei*.

While the role Yoshimune played in the development of the text is not clear, the *shōgun* responded extremely well to the work. The *Lisuan Quanshui* – which Takebe participated in translating and heavily praised the *shōgun* in – was not actually imported to Japan until 1726 CE, meaning that Takebe was assigned work on the *shōgun*’s calendar reform (a project the *shōgun* particularly favoured) after the *Tetsujutsu Sankei* had been produced. It is also the case that Takebe continued to rise up the ranks of the *shōgunate* office after the producing of the *Tetsujutsu Sankei*, eventually becoming part of Yoshimune’s inner circle and receiving honorary titles such as *yoriai* (advisor) and *hoi* (knight).[139] Takebe was even sent on a mission to discuss science with Dutch traders at Nagasaki in 1727 CE for the country’s leader.[140] By the time he retired Takebe “received a life annuity of 300 *pyo*”, which was “the same as a landlord of a 300 person village”.[141]

Terminological expressions of *ri*

One of the particular ways in which Takebe’s close relationship with Yoshimune can be seen to impact his work in the *Tetsujutsu Sankei* is in his specific, overt choice of Neo-Confucian terminology.

[138] Horiuchi, op. cit., p. 208
[139] Jochi (1997), op. cit., p. 2078
[140] Horiuchi, op. cit., p. 228
[141] Jochi (1997), op. cit., pp. 2077-8

While other Edo period mathematicians as mentioned were also known to consider the term *ri* in a mathematical context, Takebe used a different term to express the concept mathematically. The way other mathematicians had considered *ri* with respect to mathematics was as follows:

> For mathematicians of the Edo period, grasping or clarifying the principle of a method meant understanding how it worked. To go against the principle meant having an erroneous understanding of a thing or betraying its nature.[142]

The influence of Neo-Confucianism here is evident in the discussing of the clarifying of principles. This means that there was indeed consideration by other mathematicians of this particular concept. However, this conception makes no mention of attaining the *ri* through research by means of numbers, indicating some difference to Takebe's understanding of it.

Takebe's conception of *ri* also differed terminologically to other mathematicians. For example, in the writings of Takebe's older brother Takebe Kataaki 建部賢明, whom he studied mathematics with under the supervision of Seki Takakazu, there are references to *ri*. However, Kataaki's way of expressing it differs. In one passage for instance Kataaki describes mathematics as an art and tells us his brother "was extremely intelligent and had gained a deep knowledge of the Way that unifies the mathematical principles (*sūri ikkan no michi*)".[143] What is important here is that while this passage references the 'principle' *ri* in a mathematical context, as seen in the *Tetsujutsu Sankei*, the term used by Kataaki for 'principle' is not *ri* but

[142] Horiuchi, op. cit., p. 265
[143] See Horiuchi, op. cit., p. 117

sūri 数理.[144] This is a word which combines a common *kanji* for mathematics, 数, with the *kanji* for *ri* 理.

Kataaki's use of the term *sūri* rather than *ri* to describe a form of *ri* specific to mathematical discussion is important. Kataaki's writings date to around 1715 CE, seven years before Takebe wrote the *Tetsujutsu Sankei.*[145] This means that at the time Takebe wrote this text he would have been aware that when *ri* was applied to mathematics it could be expressed as *sūri* to show it was being used in a mathematical context. Given that Kataaki studied mathematics with his brother, Takebe would surely have had knowledge of this term. Also, because Kataaki most likely learned this expression while studying mathematics with his brother at Seki Takakazu's famous school of mathematics it seems likely this was a more common term among mathematicians than *ri* by itself.[146] This means Takebe was very likely aware of this expression and its use to term the *ri* in mathematical contexts by his peers, but even so consciously chose to use the exact term used by Neo-Confucians – *ri*.

While it is the case that these terms are similar, with one meaning 'mathematical principle' rather than just 'principle', Takebe's discussion and specific use of *ri* rather than *sūri* in the *Tetsujutsu Sankei* is significant. It means his mathematical language was in between Neo-Confucianism and "the technical language of a mathematician".[147] This indicates that rather than being produced for other mathematicians (for if so he would have adopted their standard technical language) the specific or primary target audience of the *Tetsujutsu Sankei* was more

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[144] In the modern day, *sūri* translates to mean 'mathematics' or 'mathematical', though in the Edo period its meaning was 'mathematical principles'.
[145] See Horiuchi, op. cit., p. 116
[146] Horiuchi, op. cit., p. 119
[147] Horiuchi, op. cit., p. 272

likely Neo-Confucians. For, in using the term *ri*, Takebe's writings do become more Neo-Confucian, as the term *sūri* determines the context as specifically mathematical. All of this indicates that Takebe was purposefully making his discussion of methodology more Neo-Confucian than was traditional for mathematicians at the time.

Takebe's conscious use of *ri* over *sūri* and inclusion of Neo-Confucian inspired methodology was most likely due to the *Tetsujutsu Sankei* being funded by the Neo-Confucian *shōgunate* and read by the *shōgun*. Also telling is the fact that the *shōgun* seems to have responded well to the text, as evidenced in Takebe's further assignment of additional projects and rising in rank. Takebe's mathematics in the *Tetsujutsu Sankei* can thus be understood as highly individualistic and as diverging from the standards of the small mathematics community he was a part of due to context. For, it was impacted and shaped by his relationship with the *shōgun* (who was personally reading his works and a Neo-Confucian) and his position in the *shōgunate* office.

Kanbun Language

Another way in which Takebe's mathematics can be seen to be impacted by his position in the *shōgunate* office and relationship with the *shōgun* is in his choice of writing. It is the case that Yoshida Mitsuyoshi wrote his mathematical work the *Jinkōki* in everyday Japanese, while the *Tetsujutsu Sankei* of Takebe was instead written in the more complex and academic *kanbun* language. This difference in language was very likely determined by his work being specifically written for the *shōgun* and the government.

The *kanbun* language was the traditional language of literature and classics in Japan. In Japanese, adopted Chinese characters known as *kanji* are used to express many words. *Kanji* can have what are known as *on* and *kun* readings where *on* readings are often the "Japanese versions of the Chinese pronunciations that were introduced into Japan" and *kun* are "Japanese words with meanings similar or identical to those of their associated *kanji*".[148] In *kanbun*, Chinese characters and grammar are used but it is often the Japanese *kun* reading for the characters which are employed, making the language outwardly display as Chinese but be read as Japanese.

From the middle of the seventeenth century, *kanbun* "formed the basis for Tokugawa…education".[149] The increased use of this language "reflected the influence of Confucian education and thought…since Confucian thought inspired many measures taken by the new government, this in turn helped perpetuate the necessity for reading and writing in *kanbun*".[150]

Being "the language of scholarship" and having correlations with Confucianism, it would have been more appropriate for Takebe to use this language to present his work to the *shōgunate* and *shōgun.*[151] The *shōgun,* after all, was the highest official in the country and highly educated. *Kanbun* was "not easy…then to read by those without a classical education", meaning that by using this language Takebe was restricting access to his work to educated officials and *samurai.*[152] Because of this, Takebe may have also used *kanbun* as a means to disassociate his work from the utilitarian mathematics of merchants.

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[148] AJALT, *Japanese for Busy People II*, p. 10
[149] Kazuki Sato, 'Same Language, Same Race: The Dilemma of *Kanbun* in Modern Japan', *The Construction of Racial Identities in China and Japan*, p. 118
[150] Margaret Mehl, *Private Academies of Chinese Learning in Meiji Japan: The Decline and Transformation of the Kangaku Juku*, p. 26
[151] Ibid.
[152] Louis M. Cullen, *A History of Japan, 1582-1941: Internal and External Worlds,* p. 127

Takebe's use of this language rather than everyday Japanese thus marks one of the ways in which his work was shaped by context. It also illustrates again how different contextual factors influenced different mathematicians in the Edo period. Yoshida Mitsuyoshi, for instance, was influenced in his use of everyday Japanese by the economic climate at the time he lived and the need of the everyday people for instructional mathematics. Takebe on the other hand can be seen to be responding to different factors such as the fact that his work was commissioned by and written for the country's elite who were highly educated. It was important for Yoshida that his work could be used by people of all classes throughout the country, while Takebe's work was intended primarily for the *shōgunate* and restricted to access by highly educated officials and persons.

Summary

Due to Takebe's position in the government and relationship with the *shōgun* Yoshimune, the *Tetsujutsu Sankei* can be understood as a supra-utilitarian work with Neo-Confucian influence. Its audience, rather than being merchants and farmers, was the country's elite and even the *shōgun* himself who was a Neo-Confucian. It was due to Takebe's position as a mathematical and scientific advisor to the *shōgun* that the *Tetsujutsu Sankei* contained Neo-Confucian concepts such as the *ri* and 'investigation of things' through immersion to express methodology. For, Takebe had to present work that was in harmony with the *shōgunate*'s belief systems and which his employers would be capable of understanding (particularly the *shōgun*). This also explains why the text was written in *kanbun* rather than everyday Japanese, for Takebe's work was commissioned by and for the most highly educated individuals in the country. Due to this the content, style, and language of the *Tetsujutsu Sankei* can be seen to be dependent upon Takebe's career, relationship with the *shōgun* Yoshimune, and the *samurai* status which enabled him to enter government.

2.4. SUMMARY

In this chapter, some of ways in which Takebe's text the *Tetsujutsu Sankei* was context-dependent were explored.

One of the great influences to shape his work was Neo-Confucianism, which was heavily based off the teachings of the Chinese Confucian Zhu Xi. Neo-Confucianism became a popular belief system in the Edo period, with this popularity largely the result of "support by leading members of the Tokugawa family".[153] Because of this favouring and promotion of Neo-Confucianism amongst the leaders who employed him, it is no surprise that Takebe shaped his mathematical methodology largely around this belief system. Because of this he also specifically used terminology which was more weighted with Neo-Confucianism than other mathematicians to express his mathematical ideas.

As this chapter showed, the circumstances surrounding Takebe vastly differed to Yoshida Mitsuyoshi, and due to these differences his work was shaped in different ways. For example, while Yoshida was free to respond to the concerns of the everyday people Takebe had a specific audience whom he had to write for due to being an employee of the *shōgunate* office and paid to do mathematical and astronomical work for the *shōgun*. Takebe also had close contact directly with the

[153] De Bary, Gluck, Tiedemann, op. cit., p. 75

shōgun himself. Because of this, Takebe was more restricted in what he could write and how he could present his work. He wrote his text the *Tetsujutsu Sankei* with the knowledge it would be read by the country's most elite rather than everyday citizens. He thus did not have the freedom of language, philosophy, and audience that Yoshida possessed. It was because of these unique circumstances that his work took the form it did as a highly specialised, supra-utilitarian, philosophical, and Neo-Confucian treatise rather than a commercial, utilitarian manual.

CHAPTER 3

EDO PERIOD CONTEXT AND THE SANGAKU TRADITION

INTRODUCTION

During the Edo period there developed a tradition in which mathematical, competitive, supra-utilitarian, religiously significant, and aesthetically orientated wooden tablets were hung in Buddhist temples and Shinto shrines. These tablets were known as *sangaku* 算額.

While there are only around 900 tablets in existence today, records indicate that during the Edo period thousands more were produced throughout the country.[154] The oldest surviving tablet dates back to 1683 CE, though the Edo mathematician Yamaguchi Kanzan was known to have referenced an older work created in 1668 CE.[155]

The content, presentation, and purpose of the mathematics on these tablets differed from both the work of Yoshida Mitsuyoshi and Takebe Katahiro. For instance, most *sangaku* appear to have been mathematical challenges which presented geometrical problems and laid the burden of proof on the observer.[156] The mathematics of these problems was also often of a relatively complex nature, to the

[154] Rothman and Hidetoshi, op. cit., p. 9
[155] Ibid.
[156] Some *sangaku*, as will be discussed in this chapter, do not focus on geometrical problems or contain visual geometrical representations of the problems. However, the majority of *sangaku* dealt with geometrical problems and had representations, and it is these which are most commonly associated with the *sangaku* tradition.

extent that some might "stop a graduate student in his or her tracks" let alone an Edo period citizen.[157]

Sangaku contained mathematics of a supra-utilitarian nature. But rather than dealing with specific subjects such as π, arcs, or mathematical techniques, *sangaku* generally dealt with determining metric relationships between various mathematical figures. Values pertaining to the problems were usually provided either in the Japanese positional notation system or using *sangi* rod depictions. Sometimes *sangaku* were authored by individuals, while others were the work of groups. There were also some which were anonymous, though a few included inscriptions detailing the date, author, and sometimes even reason for creation.

Another noticeable difference to other mathematical works was the medium and location of *sangaku*. Rather than being written in manuscript form and kept in libraries or homes, *sangaku* were inscribed on large flat wooden tablets which ranged in size but were usually of around 21cm to 72cm high and 61cm to 141cm wide. They were also displayed in prominent areas of Shinto shrines and Buddhist temples, such as the eaves. *Sangaku* were highly individualistic in nature, for each tablet was unique and had a fixed location in these religious sites.

The phenomenon of hanging mathematical works in temples and shrines was not localised to one region. *Sangaku* appeared in various locations all over Japan in the Edo period, some as grand as the Gion shrine of Kyoto, others small and rural. Their placement however was always in locations of religious significance, and the practice of dedicating mathematics in religious sites was not carried out by Yoshida, Takebe, or even Chinese mathematicians.

[157] Rothman and Hidetoshi, op. cit., p. 89

Another distinctive feature of *sangaku* was their inclusion of colourful aesthetic illustrations accompanying the problems presented on the tablets. Although pictorial representations can be found in the work of other mathematicians (such as Yoshida Mitsuyoshi's *Jinkōki*), the illustrations displayed on *sangaku* were more deliberately aesthetic. For instance, they were brightly coloured and sometimes even contained full graphical scenes that seemed more like paintings than mathematical works. However, not all *sangaku* included these colourful depictions, with a minority containing black and white illustrations, and some lacking pictures entirely. However, though some *sangaku* can be seen to differ in presentation "depending on the geographic area where they exist", they all shared a similar location and style.[158]

The purpose of *sangaku* tablets seemed to differ from other mathematics of the period. This is because they seem to have functioned as objects of exhibition which allowed mathematicians to display their work in the disconnected Edo mathematical environment. For, it was the case that "each wasan school formed a closed society", and did not generally communicate with one another.[159] As well as this, the challenging nature of *sangaku* appears due to a sense of competition amongst practitioners, and the desire of each to display their virtuosity and increase their reputation as a skilled professional.[160] The placement of these tablets in Shinto shrines and Buddhist temples indicates that they may have also severed a religious function as objects of dedication and worship.

Though some *sangaku* were created during the same era in which Takebe produced work, they did not reference Neo-Confucianism. In fact, the location of

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[158] Antonieta Constantino, *Sangaku*, p. 6 (translation from Portuguese)
[159] Ueno, op. cit., p. 75
[160] Shigeru Nakayama (1969), *A History of Japanese Astronomy: Chinese Background and Western Impact*, p.156

sangaku in sacred places of worship can be seen to connect them to Shinto and Buddhism, of which there is no parallel in Takebe's or Yoshida's mathematics. *Sangaku* also did not seem to have been designed as research treatises, as they were brief, took the form of challenges, and did not present and explain new mathematical findings. But, neither were they designed as instructional utilitarian pieces, for they did not seem to contain references to commercial or agricultural mathematics. Thus they were distinctly different to the work of both Yoshida and Takebe.

As can be seen, there are many ways in which *sangaku* tablets differ from other examples of Edo period mathematics. For example, *sangaku* seemed designed to challenge, inspire awe, and raise ones status rather than to teach or display new mathematical findings. These differences were likely shaped by contextual and environmental factors which were not significant to the same extent for other practitioners. *Sangaku* tablets thus evidence another way in which mathematics manifested itself in different ways through different people in the Edo period.

In this chapter, some of the ways in which *sangaku* were shaped by contextual factors will be examined. These include such factors as the *ema* tradition, Shinto mythology, and the *idai* tradition. It will be shown how some of the ways in which they were dependent were at times similar to other practitioners (such as in their language). However, other aspects of *sangaku*, such as their medium and location, seem caused by different factors altogether.

3.1. Ema Tradition

One feature of *sangaku* as discussed which was not shared by the works of Takebe and Yoshida was their presentation on wooden tablets and placement in Shinto shrines and Buddhist temples. It is the case that in Japan a long running "custom of hanging tablets at shrines… centuries before *sangaku* came into existence" was practiced.[161] *Sangaku* tablets were, as shall be seen, shaped by this tradition in their presentation and style.

The tablets mentioned above which were already commonly hung in shrines were known as *ema* 絵馬 – where *e* (絵) means picture or painting and *ma* (馬) a horse. In ancient times, "shrines used to keep live horses for ceremonial purposes" because they were considered messengers of the Japanese gods known as *kami* 神.[162] However, due to the expense of regularly sacrificing horses "paintings executed on flat wooden surfaces…were created as a modern substitute".[163] It was particularly during the Muromachi (1333 – 1573 CE) and Edo periods that these developed into "free-standing flat boards".[164]

In the modern day *ema* have developed to become small wooden tablets which display pictures of various animals (generally of the zodiac) on one side and have a blank area on the other (to write one's wishes to the gods). These are hung in specific areas of shrines and temples, and are burned by the acting priest when there is a sufficient number.

Ema tablets also evolved in another manner as "from the eighteenth century onwards it became a popular practice to make donations of votive tablets (*ema*)

[161] Rothman, op. cit., p. 85
[162] Elizabeth Kiritani, Vanishing Japan: Traditions, Crafts, & Culture, p. 116
[163] Ibid.
[164] Ibid.

carrying pictures of such episodes as the rock-cave or Susanowo's slaying of the *orochi* monster. Such tablets were then displayed in a conspicuous place at the shrine, sometimes even in a specially constructed *ema* hall".[165] Thus along with smaller *ema*, from around the 1700s CE onwards larger wooden tablets depicting mythological scenes also developed and were hung in prominent areas of shrines.

Image 6 - Modern day ema at Kiyomizudera Temple in Kyoto

Sangaku and Ema

Sangaku tablets had a similar constitution to *ema*. The *sangaku* depicted below from the small rural Katayamahiko shrine 片山日子神社 in Osafune, Okayama for example is very typical of most tablets. Dedicated in October 1873 CE, the tablet is constructed out of a flat wooden board which is 180cm high and 182cm wide. It contains sixteen problems, each of which have a colourful accompanying illustration. The story is that each problem was produced and dedicated by a different

[165] John Breen and Mark Teeuwen, *A New History of Shinto,* p. 162

student of a private local school of mathematics. The *sangaku* has a prominent place inside the shrine, being clearly visible to visitors ancient and modern.

This *sangaku* is visibly similar to the second kind of *ema* tablet mentioned above. It is made of wood, has colourful painted illustrations, is large in form, and also has a significant placement in the shrine. However, rather than showing animals or mythological scenes, this tablet instead contains mathematical problems and geometrical illustrations.

Image 7 - Sangaku at Katayamahiko shrine, Okayama

There are some examples of *sangaku* however which have a heavier aesthetic element than that from Katayamahiko. For example, the tablet below from Fukui prefecture does not focus on geometrical pictures and instead contains a painted scene which depicts a cherry blossom viewing party (a tradition still very commonly practised in the modern day and of cultural significance). This particular tablet, in having a graphical scene rather than a mathematical illustration to accompany the problem, seems situated somewhere in between the Katayamahiko *sangaku* and an *ema* tablet. There are also some *sangaku* which do include depictions of animals, further suggesting an influence from *ema* tablets on *sangaku*. This also illustrates how there was also some presentational variation within the *sangaku* tradition itself, which may have been due to some *sangaku* being more shaped by the *ema* tradition than others.

Image 8 - Sangaku from Fukui which appears to depict cherry blossom viewing[166]

While *sangaku* can be seen to be built upon the religiously significant *ema* tradition, the function of these tablets do seem to differ. Many *sangaku,* for instance, specifically take the form of challenges, meaning that not all tablets were treated as passive depictions or dedications.

[166] Copyright Dr Hiroshi Kotera; http://www.wasan.jp/fukui/isibe.html

For instance, Fukagawa Hidetoshi believes that while *sangaku* were "left as gifts to the gods" like *ema* it is the case that the mathematicians who produced them were also "showing off and challenging others to work out the proof" of their problems.[167] This indicates there was a social element to the tablets which did not occur in the *ema* tradition.[168] However, the proofs in many cases seemed to have been extremely difficult to solve, and some mathematicians even provided incorrect figures and results on them. In these instances, the *sangaku* may have been purposefully designed to stimulate a contemplation of the divine in the ordinary people. However, even given this, while it is evident that *sangaku* were impacted by the *ema* tradition (and some perhaps adopted their religious or mystical purpose) it is the case that their function cannot be said to be exactly the same. For, while being shaped by these tablets, they also appear to have had a more social element in their challenging contents.

Motivations

As shown, the format and location of *sangaku* were shaped by the existing tradition of hanging *ema* tablets in shrines and temples. But it cannot be said that *sangaku* are another form of *ema*, for not all *sangaku* are pictorial and their content indicates additional or differing functions. But, why did mathematics evolve in this manner if practitioners intended their work to be more of a challenge than a religious offering?

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[167] Dennis Normile, '"Amateur" Proofs Blend Religion And Scholarship in Ancient Japan', *Science;* Mar 18, 2005, p. 1716
[168] Ibid.

It is the case that in Japanese society Shinto shrines and Buddhist temples have traditionally played a significant role in the lives of communities. For example, it is at Shinto shrines that many indigenous Japanese deities known as *kami* are housed. To appease these gods and seek good fortune from them, shrines and temples are frequently visited for prayer by members of the community. As well as this, many ceremonies and rites of passage are conducted in these locations, such as birth ceremonies and weddings. Shrines and temples also have annual festivals which involve the whole community, and at special times of the year visits to these locations are common and expected (such as New Years).

Shinto shrines and Buddhist temples thus have traditionally functioned as both community locations as well as religious ones. Because of this fact, mathematicians may have specifically sought to have their work displayed there. For, it is the case that outside of specific mathematical schools there did not exist any real outlet for mathematicians to exhibit their work. These schools were also disconnected from one another and each “formed a closed society”, meaning that mathematicians were not easily able to share their work with other practitioners outside their own school.[169] By hanging work in shrines and temples, mathematicians were able to overcome this problem. It is also the case that mathematicians “formed a competitive ‘status group’” during this time, and due to this likely did seek out ways they could show off their mathematics.[170] Because of their function as community locations, temples and shrines were the perfect location to place mathematical work. By hanging their mathematics in these sites, practitioners were able to reach the widest audience possible and compete with mathematicians from different schools.

[169] Ueno, op. cit., p. 75
[170] Nakayama (1969), op. cit., p. 156

It is the case that by making the tablets outwardly visible to *ema*, mathematicians would have been more likely to be permitted to have their work displayed in shrines and temples. For, these were locations of religion, and *ema* were exemplars of the form which temple and shrine offerings took. Thus by mimicking *ema* tablets the mathematics developed a connection to worship – be it truly intended or not – and was more appropriate for display in these religious and communal sites. Also, because of the *ema* tradition already existing "it would not have seemed extremely strange to hang a mathematical tablet in a temple".[171]

Summary

In this section it was shown how the format and presentation of *sangaku* were connected to the *ema* tradition and their placement in shrines and temples. *Sangaku* follow a similar pattern to *ema* in being painted or inscribed on wooden tablets and displayed in religious locations. However, they were likely placed in these locations partly for non-religious purposes, for mathematicians used these communal sites of frequent visitation to present their work to the widest possible audience – including rival mathematicians. The degree to which their purpose was partly religious seems to have varied however, for in some instances they appear more similar to *ema* than others.

Because of these facts, *sangaku* can be understood to be dependent upon the prior *ema* tradition. They were thus influenced by different contextual factors to Yoshida and Takebe in respect to their form and location, and evidence another separate instance of mathematics manifesting itself in Edo Japan.

[171] Rothman and Hidetoshi, op. cit., p. 9

3.2. CIRCLES AND SHINTO

In the Japanese tradition circles have at times historically had religious and cultural significance. For instance, a circle plays a key role in the creation myth of the indigenous gods and the Japanese islands. Also, a circular object is part of the central myth of the sun goddess *Amaterasu*天照 who is often associated with a circle and claimed to be the ancestor of the Japanese imperial family. It is also the case that many ritualistic stone patterns in the form of circles (believed to have been for community ritual and burial) have been unearthed by archeologists which date back to the Middle and Final Jōmon periods (3000 – 300 BCE).[172]

It seems that circles may have possibly had a religious or cultural role in the *sangaku* tradition as well. For instance, an overwhelming majority of tablets deal with and depict mathematics pertaining to circular geometry problems. While there may be many possible explanations for this (as circles are cross-culturally common objects of study) this particular focus, their location in places of religious and cultural significance, and the fact that circles were not prevalent to the same extreme in the work of Yoshida Mitsuyoshi and Takebe Katahiro indicates that the heavy inclusion of circles on these mathematical artifacts may have been caused by their location or function partially as objects as worship. Given this, the following section will examine whether *sangaku* may have been partly influenced by the strong cultural and religious connection to circles which existed in Japanese society.

[172] Daisei Kodama, 'Komakino Stone Circle and Its Signficance for the Study of Jomon Social Structure', *Senri Ethnological Studies 63,* p. 235

Creation Myth and Principle Goddess

Circles, as mentioned, were connected to myth and religion in Japan. Circles were referenced with religious connotations for instance in the creation myth of the *Nihongi* or *Nihon Shoki*日本書紀 (the second oldest chronicle of Japanese history dating back to 720 CE). In this myth, the two gods who embody the female and male (or *yin* and *yang*) in the Japanese tradition (*Izanami* and *Izanagi*) meet by walking in a circle.[173] They did so as an act of courtship, and through walking in this circular manner produced offspring who became the mythological *kami* and Japanese islands.

The circle enters into mythology again in the myth of the Sun Goddess *Amaterasu* who was the daughter of *Izanami* and *Izanagi. Amaterasu* has traditionally been a particularly important deity for the Japanese people, and is considered "the chief divinity of Shinto".[174] It is believed the mythical first emperor Jimmu Tenno 神武天皇 (whose supposed rule was from 711 – 585 BCE) was the great-great-grandson of *Ninigi* 瓊瓊杵, who was the grandson of *Amaterasu.*[175] Due to this supposed direct ancestry with *Amaterasu* the Japanese imperial family were considered divine up until after World War II.

The circle is very much associated with *Amaterasu* in the Japanese tradition. For example, she is referenced and embodied "in the simple circle on the Japanese flag, which represents the mirror that is central to her myth".[176] In this myth, the mirror known as *Yata no Kagami*八咫鏡 was believed to be "the device by which Amaterasu-o-mikami was lured from her cave" after she had hidden herself away and

[173] David Adams Leeming, *Creation Myths of the World: An Encyclopedia,* volume 1, p. 156
[174] Patricia Monaghan, *The Goddess Path: Myths, Invocations, and Rituals,* p. 71
[175] Inoe Nobutaka, Itō Satsho, Endō Jun, and Mori Mizue, *Shinto – A Short History,* p. 32
[176] Monaghan, op. cit., p. 71

caused darkness in the world.[177] The *Yata no Kagumi* mirror is thought to house the very spirit of *Amaterasu.* While being sacred due to this association, it also gained much attention and admiration as a magical object after it was found "miraculously unscathed" after a fire 960 CE and survived two more in 1005 CE and 1040 CE.[178] The *Yata no Kagumi*, due to these myths and its connection to *Amaterasu*, "forms part of the Japanese imperial regalia" along with a sword and jewel.[179] It is the case that "Of these, the mirror is considered very sacred" to the imperial family.[180]

Mirrors were also venerable more generally in Japanese society and considered "a mystic symbol of purity".[181] They were thought, for example, to be capable of warding off evil spirits and illness because of a "belief that evil destroys itself on recognising itself".[182]

From this, it can be seen that circles were intimately connected to mythology, religion, and the imperial line in Japan. The use of a circle to reference *Amaterasu* on the Japanese flag for example indicates the importance of the circle as a religious symbol. It also shows how circles were used to visually reference aspects pertaining to religion and cultural mythology in the country.

Circular Imagery in Sangaku

Sangaku, as mentioned, largely dealt with geometrical problems (though not exclusively) that focused heavily on circles. For example, in Rothman and Fukagawa's *Sacred Mathematics: Japanese Temple Geometry*, 70 problems from

[177] Michael Ashkenazi, *Handbook of Japanese Mythology*, p. 216
[178] Nobutaka, Satsho, Jun, and Mizue, op. cit., pp. 81-2
[179] Ashkenazi, op. cit., p. 216
[180] Okakura-Kakuzo and F.S.K, 'Chinese and Japanese Mirrors', *Museum of Fine Arts Bulletin*, p. 10
[181] Ibid.
[182] Ibid.

their collection of 90 (taken from various *sangaku* tablets) contain circles. Of these, 67 are used in the problem. And, as well as, this an examination of 27 individual tablets dating between 1701 CE and 1914 CE reveals 20 containing depictions of circles.

An example of a circular based problem can be seen in the following which is taken from the Katayamahiko tablet. In it, a brightly coloured circle is painted which contains three additional smaller circles. Below this artistic depiction, the author sets the following problem for the observer: find the radius (r) of the smaller circles in terms of the radius (R) of the larger one.

Image 9 - Katayamahiko Problem

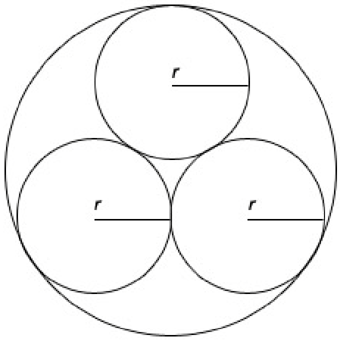

Figure 7- Modern Presentation

The mathematician writes that if $R = 10$ then $r = 4.64$.[183] However, the answer, which turns out to be the equivalent of $r = \frac{\sqrt{3}R}{2+\sqrt{3}} = (2\sqrt{3} - 3)R$, is not provided.[184] The problem is clearly a form of challenge, with the burden of proof

[183] Rothman and Fukagawa, op. cit., p. 123
[184] Ibid.

placed firmly on the observer, even though it is unlikely the average shrine patron would be up to the task of providing an adequate solution.

Many *sangaku* tablets present circular problems of this nature. Their objective seems to be to challenge the observer to calculate some proof relating to circular objects, most often pertaining to calculations or relationships regarding various radii.

This overwhelming focus on circular problems may possibly be due to the cultural and religious significance of circles in Japanese society. For as well as being inspired by *ema* tablets, the creators of *sangaku* may have further attempted to connect their work to religion and mysticism – and thus make them appropriate for placement in shrines and temples – by using circular problems. It may have also been the case that circles were used simply due to their mystical, sacred, pure, and magical connotations.

Image 10 - Sangaku from Nagasaki Prefecture containing only circular problems[185]

[185] Copyright Dr Hiroshi Kotera; http://www.wasan.jp/nagasaki/suwa2.html

However, it is the case that some *sangaku* did not contain circles. In these instances the *sangaku* usually contained colourful painted scenes of religious or cultural significance that gave them an even more similar appearance to *ema* tablets. Nonetheless, the majority of *sangaku* contained circular imagery, meaning that it seems for some practitioners one of the ways in which their work was responsive to context may have been through the use of religious and culturally significant circular symbolism.

3.3. Idai, Challenge, Status

While *sangaku* were influenced in their medium and presentation by the religiously significant *ema* tradition (and possibly the cultural significance of circular imagery), other elements of these mathematical tablets were the result of different factors. For example, their language, purpose, and sense of "challenge to other worshipers" and mathematicians seems to have also been influenced by the existing *idai* tradition the competitive nature of Edo mathematics.[186]

As discussed earlier, 'challenge'-style problems without solutions were found in the *idai* problems of Yoshida Mitsuyoshi's 1641 CE edition of the *Jinkōki*. The setting of *idai* in textbooks was to become a popular tradition in Japan during the Edo period, and sparked a spirit of challenge which found its way into other textbooks. *Sangaku* can be understood to function as types of *idai* problems due to their form as challenges. However, it is the case that their challenge was of a more difficult nature and may have even been intended to purposefully have a mystical or esoteric function.

[186] Normile, op. cit., p. 1716

Idai Inspiration

There is some evidence that authors of *sangaku* were familiar with the work of Yoshida Mitsuyoshi. For instance, a problem found on a *sangaku* in Fukutuji temple in Fukui prefecture (shown below) show similarities to a question pertaining to volume found in the *Jinkōki.* Both these problems can be seen to have the same kind of graphical depiction accompanying the problem.

Image 11 - Fukutuji Sangaku Problem[187]

Image 12 - Problem from Jinkōki

As will be recalled, the *Jinkōki* was the most popular mathematical textbook of the Edo period and found its way into most areas of society. Due to this it is highly likely that a transmission of knowledge occurred, as it seems doubtful that such a popular text would not be known to other practicing mathematicians. Also, not all *sangaku* practitioners were *samurai* or academics. Because of this, it is entirely possible that some *sangaku* were produced by farmers and merchants who were known to use Yoshida's text. Thus the mathematicians who constructed challenge-style *sangaku* mathematics most likely had knowledge of Yoshida's work – including his *idai* problems.

[187] Copyright Dr Hiroshi Kotera; http://www.wasan.jp/fukui/fukutuji2.html

Also indicating that *sangaku* may have been inspired by or designed as *idai* is the fact that the challenges set by certain *sangaku* authors seem to have been partially met by other mathematicians through the presenting of similar challenges. For example, in Hakusan Shrine, Ishikawa two *sangaku* with similar problems appear to have been dedicated around the same time.

Image 13 - 1823 CE sangaku[188]

Image 14 - 1826 CE sangaku[189]

The first, from 1823 CE, contains a problem regarding two triangles which each have a circle inscribed within them. The second, dedicated three years later in 1826 CE, also dealt with a circle drawn inside a triangle. These two *sangaku,* which are on similar topics and were devoted to the same shrine just three years apart, indicate that *sangaku* were not always passive items of worship. For, in some instances, their challenge-style mathematics inspired similar challenges.

The appearance of tablets such as those mentioned above were likely either due to certain problems being preferred by local mathematical communities or the challenges posited being addressed in the form of another challenge. This indicates a function as kinds of *idai* (at least at face value), for it is the case that *idai* inspired a

[188] Copyright Dr Hiroshi Kotera; http://www.wasan.jp/isikawa/hakusan2.html
[189] Copyright Dr Hiroshi Kotera; http://www.wasan.jp/isikawa/kasimahakusan.html

tradition in which whenever “wasan mathematicians published a book, they proposed unsolved problems at the end” to show their virtuosity.[190] In the Edo period, it was also the case that those “interested in mathematics formed a competitive ‘status group’ and created new mathematical pursuits to acquire individual fame”.[191] *Sangaku* can be seen as examples of such pursuits, with problems posited to gain fame. Thus, new challenges seem to have been presented in the same locations in competition to existing works to show how another was also a talented and capable mathematician.

Language and Status

Another way *sangaku* can be seen to be responsive to context is through their use of the *kanbun* language. There is also evidence that some *sangaku* were calculated using *sangi* rods rather than the *soroban.* Both these facts indicate that *sangaku* were connected to gaining fame and distinguishing ones work as professional.

The *kanbun* language, as has been discussed, was originally used to make “Chinese texts read like Japanese”.[192] Yoshida Mitsuyoshi wrote in everyday Japanese due to a want to make his mathematics as accessible as possible to the general populace. *Sangaku* on the other hand, which did seem also aimed at the community at large due to their location, were written in *kanbun* like the work of Takebe Katahiro.

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[190] Okumura, op. cit., p. 79
[191] Nakayama (1969), op. cit., p. 156
[192] William C. Hannas, *Asia's Orthographic Dilemma,* p. 32

It was the case that many mathematicians were discontent with the connection between mathematics and commercial activity that developed due to the popularity of the *Jinkōki.* The *samurai* particularly “despised the plebeian *soroban,* and the guild of learning sympathized” through choosing to use the *kanbun* language and *sangi* rods over the *soroban* for calculation.[193] There are some *sangaku*, such as that depicted below (*image* 14) from Gifu, which used *sangi.* However, some mathematicians, such as Takebe, did use the *soroban* for calculation, meaning that the use of *sangi* amongst professionals was by no means universal.[194] Due to the fact that not all *sangaku* depict *sangi* rods, it is likely that *sangi* were not universal amongst *sangaku* practitioners. Because of this, using and depicting *sangi* rods may have been a way to show one was especially educated.

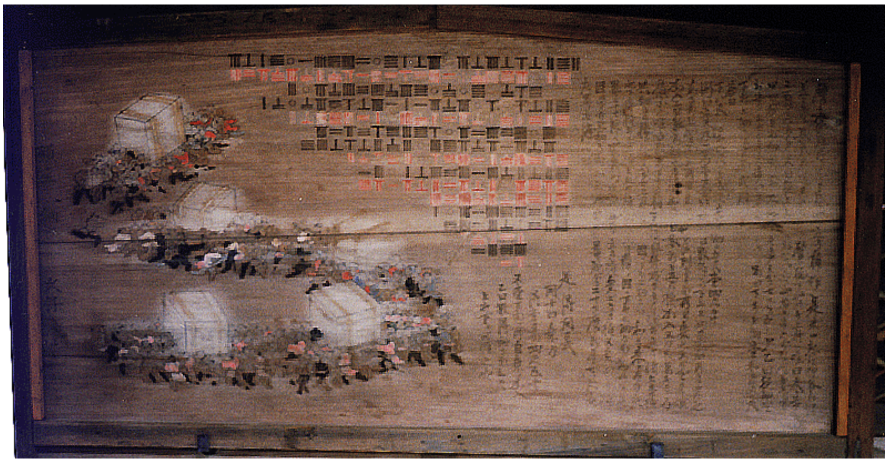

Image 15 - Sangaku from Gifu depicting sangi rods

at the top in red and black[195]

[193] Smith and Mikami, op. cit., p. 47
[194] Rothman and Hidetoshi, op. cit., p. 306
[195] Copyright Dr Hiroshi Kotera; http://www.wasan.jp/gifu/yuba.html

The use of the *kanbun* language and *sangi* rods by some *sangaku* practitioners can be seen to potentially due to the growing desire amongst mathematicians to disconnect their work from utilitarian mathematics. *Sangaku,* after all, did not deal with utilitarian problems, with the mathematics being highly abstract. Combined with the fact that *sangaku* contained challenges which as shown earlier were used by Yoshida to distinguish good from bad mathematics and increase ones status, mathematicians can be understood to have created *sangaku* partly to establish their own mathematical work as professional. Thus they established themselves as being of a class above those who used mathematics for commercial, everyday purposes.

Sangaku can thus be understood as influenced by the *idai* problems of Yoshida Mitsuyoshi's *Jinkōki.* They were also inspired by a desire by mathematicians to increase their fame and disassociate their mathematics from more commercial work however, as seen in their use of *kanbun* and *sangi.*

But, while *sangaku* were dedicated as potential religious offerings and status symbol enhancers, some seem to have been created for other reasons entirely. For example, in an inscription on a *sangaku* hung in Kitamuki Kannon temple in 1828 CE, the mathematician writes "I appreciate my master's teachings. For his kindness, I will hang a sangaku in this temple".[196] This indicates that some *sangaku* were created and displayed simply as a sign of respect to teachers of mathematics.

Summary

As has been shown, there were unique influences impacting the writers of *sangaku*. In some instances mathematicians were driven by a desire to establish

[196] Rothman and Fukagawa, op. cit., p. 89

themselves as professionals, while at other times they just wanted to show thanks to a good teacher. All of this indicates that the factors influencing the creators of *sangaku* differed to Yoshida and Takebe, and that even individual *sangaku* mathematicians were driven at times by different factors which caused variation within the tradition itself.

3.4. Isolation

Lastly, *sangaku* can also be considered dependent upon and responsive to context in the impact the isolation policy had on their development. In the Chinese tradition, which Japanese mathematics was based off before the Edo period, *sangaku*-style mathematical tablets did not appear. Also, no tradition involving the dedication of mathematics in places of worship ever developed. Because of this, the practice and style of *sangaku* was uniquely Japanese and shows how much mathematics separated from China as a result of the isolation period in the eighteenth and nineteenth centuries.

Also, while geometry was studied by the Chinese they did not focus strongly on challenges and circular geometry. The style, medium, content, and location of *sangaku* all can be thus seen as impacted and shaped by the isolation period. For, before the isolation Japanese mathematicians only studied Chinese texts, and *sangaku*-style mathematics did not appear even though the *ema* tradition was already established. Thus the isolationist environment of the Edo period was conducive to the development of *sangaku*, and they were dependent for their development on this lack of outside influence.

3.5. Summary

In this chapter the ways in which *sangaku* can be seen as shaped by context were shown. Also, it was seen how the purpose and inspiration for the construction of *sangaku* differed to other practitioners of the era.

The location of *sangaku* can be understood as impacted by a desire by practitioners to present their work to audiences. As well as this, their style as *ema* tablets was due to the tablets needing to conform and fit with their religious environment (and possibly to make them objects of worship too). The heavy use of circles on *sangaku* can too be seen to correspond to the religious location of the works.

The appearance of challenge-style mathematics on these tablets can be understood as inspired by the *idai* tradition to a certain extent. In conjunction with their difficult nature and the general lack of direct mathematical responses (for responses usually took the form of new challenges), an element of esotericism and mysticism can be seen as well.

The *sangaku* practice was original to Japan, with mathematics not taking a similar form as objects of worship and challenge in Chinese shrines and temples. This means the isolation period was also influential and conducive to this development in mathematics. *Sangaku*, lastly, were shaped by a desire by mathematicians to establish themselves as professionals and to raise their status. This can be seen in their use of the *kanbun* language and at times *sangi* rods.

Thus, different factors were important for *sangaku* mathematicians, and these differences present in the form, content, style, and location of these particular mathematical works. It is because of these factors that these examples of mathematics differed to that of Takebe, who also produced mathematics during the same era. Thus this tradition, along with the work of Yoshida and Takebe, illustrates how mathematics was shaped by context and manifested itself in different ways through different people in the Edo period.

CHAPTER 4

EDO MATHEMATICS, CONTEXT, AND PLURALISM

It has been shown how due to influence from contextual factors the mathematics of the Japanese was connected with culture, religious-laden, and highly individualistic. Also, in some instances the mathematics presented as utilitarian, while in others it appeared more distinctly supra-utilitarian. Due to this, Edo mathematics was diffuse. Different figures worked in largely disconnected groups and had differing ideas on the nature of mathematics and what problems or results were appropriate. However, despite this, methods, philosophy, and results similar to those found in other traditions did occur.

In this chapter, how context can be seen to have a definite impact on the development of mathematics is discussed. Also, philosophical concerns regarding what its role in this development implies will be addressed. As well as this, it is shown how the given case study of Edo mathematics indicates that social-rational dichotomisations of mathematics do not provide inadequate bases for understanding historical accounts of mathematical development.[197]

4.1. CONTEXT AND MATHEMATICS

The preceding chapters showed the undeniable impact context had on the focus, cause, and scope of mathematics in the Edo period. They illustrated not only

[197] Alvin I. Goldman, 'Knowledge and Social Norms: The Fate of Knowledge by Helen E. Longino', *Science*, New Series, p. 2148

the impact that context can have on mathematics, but the role it plays in the development of mathematics in varied and individualistic ways. For, it is the case that not all practitioners are drawn towards the same goals. Also, they often do not even have the same motivations or ideas regarding what type of mathematics and results are appropriate. However, even though context impacts practitioners in varying ways, sophisticated mathematics similar to that found in other traditions can still result.

The mathematics of Yoshida Mitsuyoshi, as will be recalled, was driven by external factors such as government enforced isolation, the alternative attendance policy, the banning of foreign books, and a period of rapid economic growth. Due to these factors, it took a form practical, utilitarian, and commercially and agriculturally applicable in manner. But, as well as this, additional factors such as the widespread popularity of his work inspired Yoshida to include additional recreational problems without answers of a more supra-utilitarian nature. Thus the kind of mathematics Yoshida felt appropriate to publish was driven by contextual factors external to the practice of mathematics. Also some of these factors even resulted in the increased sophistication of his work, seeing him develop more abstract supra-utilitarian problems.

The mathematics of Takebe Katahiro differed again due to context. Takebe was instructed in an environment where abstract mathematical work was favoured, and because of this the kind of mathematics he considered appropriate differed to Yoshida. For example, while Yoshida used the approximation of 3.16 for π because of its usefulness for general, quick calculation, Takebe instead calculated π to an impractical degree (forty-one places). Due to his considering of abstract non-practical mathematics as appropriate for mathematical study, and not worrying about the ease or applicability of his work, Takebe produced results that were beyond practical usefulness and realistic implementation for merchants and farmers.

Takebe was also influenced by his position in the Tokugawa *shōgunate* office due to Neo-Confucianism being popular among his elite employers. This meant that mathematical writings which purposefully included heavy reference to Neo-Confucianism were considered appropriate by him. However, these writings may not have been as well received by the general mathematical populous, who appeared to have used different terminology more separated from the belief system. From this it can be seen that the kind of mathematics Takebe found appropriate, and his motivations for producing it, were influenced by context. But even though this was so, these factors did not impair his ability to produce sophisticated mathematical results.

In the *sangaku* tradition, context motivated practitioners to create recreational, highly abstract geometrical problems. These were usually beyond the comprehension of average citizens, and hung in places of religious worship and significance. *Sangaku* practitioners differed again in what they thought was appropriate for mathematics. Many *sangaku* problems were of an almost arcane, mystic nature, combining mathematics and religion, and sometimes the problems presented even seemed beyond solving by the practitioners themselves. In this instance, the results were not as important as the proposing and contemplation of the problem (or new problems). This was a mindset different to Yoshida and Takebe, who valued the grasping and comprehending of mathematics.

As these examples indicate, there were different spiritual dimensions to mathematics in the Edo period as well caused in part by influence from context.

Sangaku, in being shrine offerings, having heavy circular imagery, and also an almost mystic nature were religiously significant and connected to Shinto. However, the specific extent to which individual tablets had a religious function differed within the tradition itself, with some seeming more spiritually inspired than others. The mathematics of Takebe Katahiro also had a heavy religious element, but this time from Neo-Confucianism. These examples thus illustrate the various spiritual dimensions which existed within the Edo mathematical environment.

Other examples of context influencing mathematical development can also be found in differing traditions, indicating that context influences mathematical practice cross-culturally. For example, the kind of mathematics chosen to be studied by medieval Islamic mathematicians was in some cases largely determined and driven by contextual cultural or religious factors. In the Islamic tradition for instance, “the performance of the most important ritual acts – prayer at specific times and in a specific direction, and fasting during a particular month – has been assisted by procedures involving astronomy and mathematics”.[198] With regard to prayers, which were conducted five times daily, the “problem of finding the direction of Mecca relative to a given locality” was an area that “many of Islam’s greatest scientists devoted some attention to”.[199] Their subsequent study of spherical trigonometry as a means to assist in the practice of ritual prayer was thus a “product of the religion of Islam”.[200]

For medieval Islamic practitioners, mathematics had as one of its “practical applications…the fulfillment of religious duty”.[201] Spherical trigonometry and

[198] T. Koetsier, Luc Bergmans, *Mathematics and the Divine: A Historical Study*, p. 163
[199] J.L. Berggren, *Episodes in the Mathematics of Medieval Islam*, p. 182
[200] Ibid.
[201] Victor J. Katz, *Using History to Teach Mathematics: An International Perspective*, p. 173

mathematical geography were studied with a particular function pertaining to practical religious ritual in mind.[202] This illustrates another instance historically in which external contextual factors unique to a particular culture have influenced and shaped the kind of mathematics that is studied and what is considered important and appropriate. Also, it shows another instance in which mathematics can be seen to have a spiritual dimension, and thus illustrates how context has a definite impact on mathematics.

Different Approaches, Similar Results

Even though there existed a great diversity of results and approaches in the history of mathematics – as Edo Japan and the Islamic tradition evidence – in some instances the methods, ideas, and results of various figures can be seen to be strikingly similar to those found in other traditions. This indicates that variety and contextual influence do not necessarily affect the ability of an individual or tradition to produce mature or 'universal' results in harmony with others.

For instance, there is one area of Edo mathematics that is similar to that found in Greek pertaining to Takebe's calculating of π using n-sided polygons. This technique, as shall be seen, was somewhat similar to that used by Archimedes of Syracuse.

Archimedes calculated π by "inscribing or circumscribing polygons inside or outside of the circle. As the sides of the polygon are doubled, the area of the polygon, which is always known, becomes larger and larger if it is inscribed, or smaller and smaller if it is circumscribed. Thus, whether it is inscribed or circumscribed, the

[202] Ibid.

polygon approaches the area of the circle".[203] Archimedes' approach was to create upper and lower bounds using the inscribed and circumscribed polygons, and then use a method of exhaustion in which the number of their sides were increased to the form of a circle.

Takebe's approach shared some features of Archimedes'. He used "a 1,024-sided regular polygon inscribed in a circle" to find π to forty-one decimal places.[204] Though he also adopted a method similar to Richardson extrapolation, Takebe's method can be seen to resemble Archimedes', because he also used a polygon with increasing sides to calculate π.

The form of the result Takebe gave for π was different from Archimedes however, and this difference was caused by contextual factors. Archimedes' calculation of π is considered "a remarkable achievement, since the Greek number system was awkward and used letters rather than the positional notation system used today".[205] Because of the fact that Archimedes had use of "neither decimal notation nor any other positional notation…he stated the result he obtained in terms of polygons…using fractions $3 + \frac{10}{71} < \pi < 3 + \frac{1}{7}$".[206] Because of limitations in the expressive mathematical language of the Greeks – in particular, their lack of positional notation – the approximation for π that Archimedes provided was in the form of a range that existed between the two fractions $3\frac{10}{71}$ and $3\frac{1}{7}$. For Archimedes, providing a mathematical result in the form of such a range was appropriate because the modes of expressing mathematical results available to him limited his choice of

[203] Edward Grant, *Science and Religion, 400 B.C. to A.D. 1550: From Aristotle to Copernicus,* p. 72
[204] Rothman and Hidetoshi, op. cit., p. 74
[205] S. A. Paipetis, Marco Ceccarelli, *The Genius of Archimedes – 23 Centuries Influence on Mathematics, Science, and Engineering: Proceedings of an International Conference Held at Syracuse, Italy, June 8-10, 2010,* p. 422
[206] Jesús Guillera, 'History of the formulas and algorithms for π', *Gems in Experimental Mathematics: AMS Special Session on Experimental Mathematics, January 5, 2009, Washington, Part 3,* p. 174

presentation. Therefore, due to his being part of the Greek environment, a result in the form of a decimal approximation was not appropriate or possible.

Takebe, on the other hand, provided a decimal approximation correct to forty-one places due to the Japanese number system using positional notation adopted from China. Because of this, and the fact that results in the form of ranges were not standard practice in Japan, giving as detailed a decimal approximation as possible was a suitable mathematical result for Takebe. However, Takebe may have been restricted in his ability to create higher approximations due to his use of the *soroban* for calculation.[207] This is because *soroban* only had so many rows (the maximum usually being thirty-one), and calculating numbers greater than this number would have been difficult. In producing his initial result, Takebe already was doing some "hard soroban handling", and computing even more places may have been too difficult for him to manage on the device.[208] Thus the very calculation tool Takebe employed may have impaired his ability to produce further approximations.

Takebe's providing of such a result can thus be understood to be due to contextual factors such as the prior introduction and adoption of the Chinese number system (which used positional notation) and the *soroban*. The approximation of Archimedes, which was attained using a similar method to Takebe, can be seen to differ in its form and presentation due to contextual factors also. For, as mentioned, Archimedes' lack of knowledge of positional notation – caused by the fact that the Greek environment he was part of did not yet possess it – meant his results were limited to a specific and differing presentation and form. Thus, his idea of what

[207] Rothman and Hidetoshi, op. cit., p. 304
[208] Rothman and Hidetoshi, op. cit., p. 306

constituted appropriate mathematics and practice can be understood to have been in part contextually determined.

This discussion shows that although the motivations, audience, and ideas about what kind of answers and practice were appropriate differed between Archimedes and Takebe due to contextual influences, their methods were still similar. Also, Takebe, who was part of differing social environment, produced sophisticated mathematical results even though he was impacted by different factors (such as Neo-Confucianism). This indicates that the contextual influences Takebe was subject to did not affect his ability to produce quality results pertaining to the same mathematical object of study as Archimedes, but it did cause them to be presented differently.

Another historical example of different approaches resulting in similar results can be seen in the approaches to astronomy of the ancient Indians and Greeks. The Hellenistic astronomer Ptolemy (circa 100 CE – 175 CE) for example produced an astronomical model which incorporated a series of epicycles that was inspired by the Greek conception that the "planets tended to move in circles…the circle is a perfect figure preferred by nature".[209] This model, though it conceptually lacked verisimilitude, "could make quite an extraordinary number of predictions".[210]

Indian astronomers, using the same data but with a different conceptual approach, produced predictions that were a match for Ptolemy. The Indian mathematician Âryabhata (476–550 CE) for instance, while borrowing concepts such as epicycles from Ptolemy, conceived of them differently. In his writings "the

[209] Roddam Narasimha, 'Axiomatism and Computational Positivism: Two Mathematical Cultures in Pursuit of Exact Sciences', *Economic and Political Weekly,* p. 3651
[210] Ibid.

epicycle is only used as a convenient representation of motion" and there is "no notion that the circle is a perfect figure and must therefore form a necessarily correct description for planetary orbits".[211] Without the philosophical notion of the circle as perfect, and having to 'save' the phenomena, he produced a set of algorithms rather than seeking a detailed account of heavenly motion because he found the "actual method of calculation of planetary parameters" more important.[212]

This method of the Indians Roddam Narashimha describes as 'computational positivism' due to their placement of "computation and observation at the forefront" and view that "Elaborate physical models and the process of deduction based on axioms are not…of great value".[213] Though the work of the Indians produced accurate predictions, the work of Ptolemy also produced results that were of reasonable accuracy and held in high regard in Europe for some time. This indicates how once again practitioners in different countries with different conceptions about what kind of mathematical results were appropriate could produce similar results of sophistication. For, in the case of the Greeks, results that conformed "to their ideas of perfect figures, symmetry, beauty etc" regarding circles were appropriate.[214] But it was computation combined with observation without a need for grand philosophical conceptions of heavenly motion that was acceptable for Indian mathematicians.[215]

These examples indicate that while context has shaped what was considered appropriate practice for mathematicians historically, and resulted in a variety of conceptions and approaches, mathematical results of less sophistication were not necessarily a consequence. In fact, sometimes completely different environments and

[211] Narasimha, op. cit., p. 3652
[212] Ibid.
[213] Ibid.
[214] Ibid.
[215] Ibid

factors led mathematicians to the same kind of results. As this shows, context was often influential and determining of the ways in which mathematics was developed and practiced historically.

A More Modern Example

Context can also influence the kind of mathematical results that are considered appropriate in more modern traditions. This means that the influence of context is not isolated to historical cases, and also has a definite impact on modern practice.

For example, in 'A Comparison of Two Cultural Approaches to Mathematics: France and Russia, 1890-1930', Loren Graham and Jean-Michel Kantor describe the impact of different contextual elements and factors on mathematicians in France and Russia around the turn of the twentieth century. They describe how these factors influenced certain attitudes regarding what was considered appropriate mathematics at the time set theory entered onto the academic scene.

They believe that though both sets of mathematicians had access to the same information and were considering the same problems, "the secular and rationalist culture of France worked against mathematicians' acceptance of infinite sets (in particular, nondenumerable ones) as legitimate mathematical objects, while the mystical religious views of the founders of the Moscow School of Mathematics acted as a positive influence in such acceptance".[216]

[216] Loren Graham and Jean-Michel Kantor, 'A Comparison of Two Cultural Approaches to Mathematics: France and Russia, 1890-1930', *Isis*, p. 58

The factors in France believed to have been of particular influence were "Descartes, positivism, and Pascal".[217] Cartesianism promoted thoughts such as "Every problem should be decomposed into its simple components" and that mathematics was the most "universal and least biased form of knowledge".[218] There were also influential positivistic notions of the goal of mathematics being "no longer a metaphysical quest for truth" but a set of laws and facts.[219] They believe these caused the French to feel a resistance to "mixing psychology or philosophy…with mathematics", for whom "mathematical notions were to be restricted to those for which both a clear definition and a clear 'representation in the mind' could be found".[220] Due to this kind of thinking, they feel it was more difficult for the French to accept infinite sets. For, these sets could not be clearly represented in the mind and seemed to be mixed with philosophy, being "more German metaphysics than mathematics" to some thinkers.[221]

However, in Russia there were different contextual factors at play which saw infinite sets approached more favourably, such as "mystical religious beliefs, particularly those of the Name Worshipping movement".[222] The 'Name Worshiping' or 'Nominalist' movement was a trend in the Russian Orthodoxy. It saw practitioners respond to the "question as to how humans can worship and unknowable deity" by decreeing that "the worshipper achieves a state of unity with God through the rhythmic pronouncing of his name", meaning that through the very naming of God he could become knowable.[223]

[217] Graham and Kantor, op. cit., pp. 64-5
[218] Ibid.
[219] Ibid.
[220] Ibid.
[221] Graham and Kantor, op. cit., p. 58
[222] Graham and Kantor, op. cit., p. 72
[223] Graham and Kantor, op. cit., pp. 67-8

An influential figure believed to have "played a role in the pioneering mathematical work of Egorow, Luzin, and their students" in the Moscow school of Mathematics was Pavel Florenskii.[224] He was man who embraced religion, philosophy, psychology, and mathematics and was a 'Name Worshipper'. Because of his nominalism, "Florenskii believed that both religious and mathematical symbols…attain full autonomy", and thus mathematicians "could create beings – sets – just by naming them".[225] For Florenskii, a set was "an entity named according to an arbitrary mental system, not an ontologically existing object", and this thinking meant that infinite sets were seen not as ontological objects but rather mathematical entities that came into creation upon being named.[226] Because of this, the Russians were more open to the ideas of Cantor, and proceeded to develop work with infinites and a descriptive theory of sets.

This historical discussion of Graham and Kantor illustrates how context can play a role in influencing mathematicians' approaches and the modes of mathematical practice and thinking they deem appropriate in more recent times. For, both sets of individuals had access to the same mathematics of Cantor but due to their environment and the ways context influenced itself upon them their reactions to it differed (as well as, at least initially, their results).

Another example of this is classical and intuitionistic logic, which sees logicians adopt some different standards and requirements regarding what kind of results are appropriate. For example, while the classical logician can be satisfied using the law of excluded middle to create a proof by contradiction, the intuitionist

[224] Graham and Kantor, op. cit., p. 66. All three figures were to be held accountable for their mixing of philosophy and mathematics, with Florenskii and Egorov arrested for this and Luzin being put on trial.
[225] Graham and Kantor, op. cit., p. 70
[226] Ibid.

rejects this law and does not consider a result found by this method suitable. Thus, context can be seen to impress itself – playing a role in motivating specific approaches to mathematics as well as ideas of what is appropriate – through different people historically, cross-culturally, and in more modern times.

Context, Culture, and Religion

The above discussion also reveals another point about the relationship between context and mathematics which is also supported by the Japanese and Islamic cases. This is namely that contextual factors connected to culture and spirituality can in some instances influence the development of mathematics and what mathematicians consider important in positive (or at least neutral) ways.

For example, in the Japanese tradition the work of Takebe Katahiro in the Tetsujutsu Sankei was heavily (and purposefully) influenced by the belief system of Neo-Confucianism. However, the results produced in this text exemplify some of the most advanced mathematical results of the Japanese mathematical tradition. The methodology Takebe presented can also be understood to resemble similar trends of thought in the 'Western' tradition. So, although Takebe was influenced by his environment to adopt a Neo-Confucian inspired approach to mathematics, it did not impair his ability to produce sophisticated (and, in the case of π, universalistic) mathematical results and methodology comparable to those attained elsewhere.

As well as this, the mathematics of Islam and episode in France and Russia around the turn of the twentieth century illustrate how religion can have a positive influence on the development of mathematics. They show how religion can shape the style and kind of mathematics being done – with mathematics itself even viewed with a religious element – without resulting in less sophisticated results.

This shows that while context may sometimes have what would be viewed in modern times as a negative result on mathematical development, sometimes it does in fact produce positive results. This means its influence is by no means irrelevant or trivial to mathematical development. It is the case that in the modern day "We are accustomed to think of mathematics and religion as quite irrelevant to one another", but this manner of thinking, and the separation from contextual factors it creates, may result in new positive influences being unable to develop or being undervalued.[227] Thus context can be seen to have a definite impact on mathematical development (and variations of it) in past and present mathematical traditions.

4.2. Philosophical Concerns

It has been shown how context had an undeniable role historically and cross-culturally in the shaping of mathematics and mathematical variety. Also, how this influence was not necessarily negative and produced similar and sophisticated results was illustrated. However, these discussions may raise some philosophical concerns regarding the way in which mathematics is to be viewed and approached. For instance, in recognising the heavy influence of context in shaping mathematical expression and development, must it be conceded that mathematics is a social construction? Or, rather, does the fact that similar results can be obtained in separate traditions indicate that mathematics is in fact universal, and it is the results rather than the motivations for them that should be our concern?

[227] Katz, op. cit., p. 173

It is sometimes the case that a dichotomisation of these positions occurs, where mathematics is either considered "rational and not social or social but not rational".[228] The two are also usually very much considered "mutually exclusive" as well.[229] Given this, and the discussion above, one might ask which side of this apparent dichotomy should be adopted, or how it is to be negotiated.

Also, given the importance of the role of context on mathematical practice, does this entail that the historian, philosopher, and mathematician ought to adopt pluralism, a thesis which holds that no one approach is to be "deemed to be more fundamental than any of the others"?[230]

Construction, Cultural Basis, and Context

As indicated above, it could be argued that the presented research on context and mathematics supports the conception that mathematics is a social construction or has a cultural basis.

Social constructivism, Paul Errest explains "views mathematics as a social construction. It draws on conventionalism, in accepting that human language, rules and agreement play a key role in establishing and justifying the truth of mathematics".[231] It can also be thought of, as Joseph Dauben writes, "reducing the history of mathematics to little more than a study of social context".[232]

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[228] Goldman, op. cit., p. 2148
[229] Ibid.
[230] Simon Blackburn, *Oxford Dictionary of Philosophy*, p. 290
[231] Paul Errest, *The Philosophy of Mathematics Education*, p. 42
[232] Joseph Dauben in *From Natural Philosophy to the Sciences: Writing the History of Nineteenth-Century Science*, chapter five, p. 152

In recognising the defining role that context had on the development of different approaches to mathematics and conceptions about what kinds of answers were appropriate in Japan and elsewhere, the social constructivist would argue that the very truths of mathematics for these practitioners were social constructs. That is, they resulted from social and cultural contextual factors. Thus instead of conceding that context played a role in shaping attitudes towards approaches and conceptions of mathematics, context is understood as originating them. Given this, the variety in mathematics which can be seen is solely the result of contextual factors.

For example, due to certain contextual factors Yoshida Mitsuyoshi produced a mathematical text which largely focused on instruction and how to come to understand solutions. *Sangaku* practitioners on the other hand focused not on answers but on the very act of producing work in need of proof. Instead of instructing observers they challenged them to produce proofs which were not easily attainable even for themselves. The social constructivist would argue that the contextual factors that led to mathematics being viewed in such different ways by these practitioners – one very utilitarian and the other extremely abstract – actually created these instances of mathematical activity. Thus, the mathematics of Yoshida and *sangaku* practitioners is not even the same mathematics. That is, it is does not classify as being part of the same universal external discipline of 'mathematics'. Rather, they are individual instances of mathematics given birth to by different environments and social factors.

A similar idea is that which holds that "mathematical knowledge is culturally based", and that there is no acultural or pan-cultural mathematics.[233] This view is often connected to ethnomathematicians who wish for the unique mathematics of

[233] Bill Barton, 'Making Sense of Ethnomathematics: Ethnomathematics Is Making Sense', *Educational Studies in Mathematics*, p.202

cultures such as the Japanese to be appreciated as individual instances of mathematics rather than viewed as proto or underdeveloped work.

It is the case then that by promoting the recognition of the influence of context one may be at risk of being classified a social constructivist. Social constructivism is an approach that has some unfortunate consequences, for if one is to embrace such ideas there is much about Edo period mathematics of interest that may be considered unimportant or irrelevant. For instance, the mathematics of Takebe Katahiro contained rich mathematical truths, and his recognising of the nature of π and the universality we see in his results, methods, and approaches (which at times are very similar to those of other practitioners and traditions, such as Archimedes) cannot be fully appreciated if we are to concede that mathematics is solely just the product of social circumstance.

With social constructivism failing to fully recognise the depth of Edo period mathematics (and also the thesis of universalism it can even be seen to help support), what other options are available?

Realism and Aculturalism

On the other side of the spectrum there are views pertaining to philosophy that consider mathematics as (or is best done as) acultural, transcendent, and external to humans. Commonly connected to such thoughts is the philosophy of realism, which is "the idea that the truths of mathematics are not of our making".[234]

[234] Michèle Friend, *Introducing Philosophy of Mathematics*, p. 28

Opposed to the social constructivist who holds that mathematics is just a social construction created by certain conventions or culturally based, the realist (a term which is used very broadly here, for there are different degrees and kinds of realism) sees mathematical objects and truth as external to and transcendent of humanity. Thus mathematicians uncover rather than create mathematical truths.

The realist or aculturalist about mathematics would say that the fact that Takebe independently conceived of infinite series, or that *sangaku* practitioners studied and produced geometrical studies on topics (such as circles) common cross-culturally while in an isolated environment indicates a universalism transcendent of culture. For them, certain Edo mathematicians can be seen as being at times mentally organised in such a way that was productive and allowed mathematical truths to be uncovered or accessed. Even if the presentation and purpose of mathematics such as *sangaku* geometry differed to that found in other cultures (as it did in style and location for instance), mathematicians still produced mathematics pertaining to the same common mathematical objects of study of other cultures (such as circles and arcs, etc), indicating transcendence and the existence of external truths and objects in mathematics.

For holders of such a position, the influence of context is unimportant or trivial, for all that matters is the results produced and how well they conform to or match modern methods, approaches, and philosophical conceptions. Contextual values may even be “thought to threaten the integrity” of scientific and mathematical inquiry for some.[235] What matters is that results can be seen as referring to the same mathematical objects. How they are come to, thought about, or presented is less important and at best supplementary to this.

[235] K. Brad Wray, ‘A Defense of Longino’s Social Epistemology’, *Philosophy of Science*, p. S539

There are consequences too for holding this position, as an aculturalist or realist does not see the relevance of parts of Japanese Edo mathematics which are socially rich. For instance, the practice of hanging *sangaku* in temples and shrines was of much importance for amateur and professional mathematics, because it inspired the creation of works which were of a highly abstract nature, provided an outlet for practitioners to exhibit their work, and saw their work function also as religious offerings. Also, Yoshida Mitsuyoshi's *Jinkōki*, though seeming trivial due to its containing of commercial, utilitarian mathematics, was responding to social need, caused mathematics to spread throughout Japan, and helped spark the *wasan* tradition. Further, the methodology of Takebe, which can be seen to be similar to thoughts in Europe, was only rich due to its being derived from and inspired by Neo-Confucianism. In fact, it cannot even be properly understood and translated without knowledge and recognition of Neo-Confucian philosophy and terminology. This kind of approach thus fails to fully appreciate and recognise deep social connections that often occur, and as a result may in fact impair our ability to properly understand mathematics that has been produced historically. But, if this approach is also inadequate, what position should be taken?

Moderating Dichotomisation

As shown, there are problems with both social constructivist and realist approaches to mathematics. The social constructivist is unable to appreciate richness and universality in approaches to mathematics, while the realist is unable to recognise the relevance of contextual factors of importance to past practitioners and which are sometimes necessary to even make sense of historical mathematics. As mentioned, these positions often form a dichotomy and are considered mutually exclusive. But, the deficiencies of both positions means this mutual exclusiveness should be re-evaluated and a more moderated stance inclusive of elements of each adopted.

Inclusiveness should be pursued because while it is the case that context does play a role in various modes of mathematical practice that appear, this does not necessarily mean that mathematics is a mere social construction. For while context aids in mathematical variety it alone cannot be said to be solely responsible for it, as it is due to the rich and expansive nature of mathematics to begin with that this variety in practice even occurs. There are many different areas in mathematics (even in the modern day) such as logic, topology, statistics, and set theory, and because of this natural variety different figures cannot be expected to be doing the same kinds of mathematics. However, the kind of mathematics practitioners do choose to focus on is, as has been shown, impacted or influenced by contextual factors. Thus context does play a role in the expression of this natural variety.

It is the case then that while variety in expression and practice is not solely due to context it does still play a significant role in determining the ways in which mathematics develops. This indicates that while context is not solely constructing of variety in mathematics neither is mathematics purely acultural. For cultural factors, such as Neo-Confucianism and the *ema* tradition in Japan or the daily ritual prayers in the Islamic tradition, had a real impact and influence on how mathematics advanced and on what was considered important mathematically.

This toning down of aculturalism and weakening of the mutually exclusive social-realist dichotomy that may occur does not necessarily mean the thesis of mathematical realism (and thus universalism) has to be abandoned, for as Roddam

Narasimha writes:

> …all practising scientists know that, even today, mathematics done in any country, even within the western cultural area, has its own special character (e g, British, French, Russian etc). This does not appear to be just a matter of style, but rather of philosophy: of the questions asked and the manner in which they are tackled…but some of course will be more effective than others.[236]

As Narasimha states, the variety that we see in approaches, methods, and practices in the history of mathematics (such as in Japan, ancient Greece, India, and more modern Europe) can be seen as indicative of stylistic or philosophical differences rather than individual constructed instances of mathematics whose truth depends on convention. For, as has been shown, different approaches can produce the same kind of mathematical results pertaining to the same mathematical objects. This indicates that universality in mathematics and the thesis that mathematicians uncover rather than construct mathematics and its truths is not necessarily false.

The influence and impact context has on the ways mathematics manifests shows that, as Luke Hodgkin writes, while "we do mathematics one way… we might well do it another". [237] Thus we can access the same universal mathematical objects but through different paths. So, rather than promoting an anti-universalism, context and its role in variety shows that there can merely be different degrees of "agreement with observation, economy of thought or process…phenomenological domain

[236] Narasimha, op. cit., p. 3650
[237] Luke Hodgkin, 'Mathematics as Ideology and Politics', *Radical Science Essays,* p 174

covered, or even notions of beauty or symmetry" in mathematics.[238] Thus, a blended approach that does not dichotomise contextualism and rationalism and treat them as mutually exclusive should be adopted.

Pluralism

But what does this mean for pluralism? Should the historian, philosopher, and mathematician be a pluralist about mathematics? The answer is, to a certain extent, yes. The above discussion indicates that there is pluralism evident in approaches, practices, and results in mathematics. However, although there can be this plurality of different approaches and degrees of effectiveness created by context and the natural variety of mathematics, it is also the case that there can be universality. This is because we can understand these approaches as relating to the same mathematical objects, and means it is not necessary to go to the extreme of being a pluralist about mathematical truth itself. Instead varying degrees of conformity with or realisation of mathematical truth dependent upon different approaches and styles influenced by context are recognised. Thus context, variety, and plurality in mathematics should not be ignored or treated as irrelevant or trivial.

So, while context has a significant, undeniable role in the development of mathematics historically, cross-culturally, and in modern times this does not necessarily entail social constructivism or pluralism of mathematical truth.

[238] Narasimha, op. cit., p. 3650

4.3. SUMMARY

It has been shown how context is indeed important for mathematical development and its influence is significant and undeniable. As well as this, any dichotomisation of mathematics as social or rational should be re-evaluated. For, the examples of Edo period mathematics examined (as well as additional case studies from other traditions) show that neither a purely social constructivist or aculturalist realist approach can provide a satisfactory explanatory account of mathematical activity due to their failure to capture all the intricacies of the historical cases examined.

Rather than being problematic, the moderation of such dichotomisations allows contextualism, richness of variety, and universalism in mathematics to be embraced and recognised. For context and social factors can be understood as shaping of instances of mathematics that one might call illustrative of universalism (such as the work with π done by Archimedes and Takebe). Thus it is not necessary to treat these conceptions as mutually exclusive, for both contextualism and universalism can be adopted – and even some plurality – without having to admit to aculturalism, relativism, or constructivism.

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